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Analysis of Value-at-Risk (VaR) of Naira against BRICS Currencies

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Aim: This study investigates foreign exchange market dynamics by forecasting and analyzing the Valueat-Risk (VaR) for the Nigerian Naira against BRICS currencies utilizing daily data from January 1, 2010 to December 31, 2024.

Design/Research methods: The five BRICS currencies (BRL, RUB, INR, CNY, and ZAR), were analyzed to explore the impact of foreign exchange market dynamics on the Nigerian Naira against BRICS currencies. The value-at-risk methodology was implemented plus the Monte Carlo simulation. The calculated VaR95% quantifies potential losses, emphasizing the importance of managing downside currency exchange risks in a volatile financial market at both the 95% and 99% confidence thresholds. The robustness of the Monte-Carlo simulation (MCS) and historical simulation (H-S) results validates the conditional variances and the corresponding value-at-risk estimates for the Naira exchange rate in relation to each currency of the BRICS derived from the variance-covariance model. GJR-GARCH model reveals critical insights into the valuation and volatility risk associated with the Naira exchange rate against BRICS currencies.

Findings: The valuation of the Naira/Real rate has significant vulnerabilities to changes in oil prices, external debt, and changes in money supply; the results show that the Naira/Rubble exchange rate had significant and negative responsiveness to changes in output growth, crude oil prices and external debt levels; valuation of the Naira/Rupee exchange rate is significantly responsive to the vulnerability of trade balance, external reserves, foreign debt, monetary policy rate, and crude oil prices; valuation of the Naira/Yuan exchange rate has significant vulnerabilities to changes in oil prices, output growth, external debt, and CBN policy rate; valuation of the Naira/Rand has significant vulnerabilities to changes in external reserves, financial healthiness and external debt levels.

Originality / value of the article: The study findings are robust explanation of asymmetric risk identified by VaR with policy advice for the CBN to strategically rebalance its exposure to BRICS currencies by using risk-weighted analysis instead of just trading volumes. Also, the study contributed to prediction of possible losses associated with unfavorable Naira currency fluctuations when trading particularly with

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the BRICS, and so emphasized the necessity for the adoption of VaR-based stress testing to national foreign exchange reserves and financial institutions. Conclusion: Given the asymmetric risk, the CBN should intentionally rebalance its exposure to BRICS currencies by using risk-weighted analysis instead of just trading volumes. For example, the CBN ought to look at establishing more local currency settlement agreements with the BRICS countries. This could reduce exposure to the volatility of the US dollar and dependence on it.

Keywords: Variance-Covariance methodology, Monte-Carlo Simulation (MCS), Historical-Simulation (H-S), BRICS currencies, VaR, Naira Exchange Rate *JEL: B23, D25, C17.*

1. Introduction

The performance of the Nigerian Naira in the foreign exchange market has been marked by substantial volatility, particularly when compared to other global currencies. This volatility is primarily driven by Nigeria's heavy reliance on oil exports, which leaves the Naira vulnerable to fluctuations in global oil prices. Between 2014 and 2016, a significant decline in oil prices contributed to the weakening of the Naira as Nigeria's foreign reserves dwindled, forcing the Central Bank of Nigeria (CBN) to adopt measures aimed at stabilizing the currency (FEWS NET 2016). Thus, to address this issue, in 2017, the CBN introduced the Investors and Exporters (I&E) window to facilitate a market-driven exchange rate, thereby improving liquidity and reducing the pressure on the official exchange rate. Despite these efforts, the Naira continues to experience substantial fluctuations, driven by domestic policy and external factors, such as global economic conditions and speculative market activities (Chinwe 2024). Additionally, the continued presence of the parallel market, where exchange rates often reflect a more accurate market-driven value of the Naira, has exacerbated the discrepancies between official and market rates, complicating the management of the Naira's value (Bamidele 2024). So, the Naira's ongoing volatility remains a critical challenge for businesses and consumers in Nigeria, influencing inflation, trade, and investment decisions. The complexities of managing the Naira's value in this dynamic environment state the importance of consistent and adaptive economic policies (Olu, Anu 2019).

This research centers on understanding the Value-at-Risk (VaR) of the Naira exchange rate vis-à-vis the currencies of the BRICS countries. This is crucial for

ANALYSIS OF VALUE-AT-RISK (VAR) OF NAIRA AGAINST BRICS CURRENCIES

policymakers, investors, and businesses in those nations engaged in international trade and investment. Among the BRICS nations, the Brazilian Real (BRL) has limited direct impact on Nigeria's economy but plays a role through trade in commodities like agriculture and raw materials. Additionally, Brazilian investments in Nigeria's infrastructure and mining sectors create indirect linkages between the BRL and the Naira (Fernandes 2024). The Russian Ruble (RUB) influences Nigeria primarily through energy, defense, and mining industries. While its direct economic linkages are small, fluctuations in the RUB-Naira exchange rate impact the cost of Russian imports, including oil and military equipment (Ezinwa, Anyanwu 2019). The Indian Rupee (INR) influences Nigeria's economy through trade in pharmaceuticals, technology, and other manufacturing sectors. Economic or currency fluctuations in India can affect Nigeria's import costs and capital flows, while India's growing investments in Nigeria in sectors like technology and education further connect the two economies (Oseni 2020).

Moreover, the Chinese Yuan (CNY) is perhaps the most significant BRICS currency for Nigeria. China's status as one of Nigeria's largest trade partners, especially in non-oil goods like electronics, machinery, and manufactured products, means that fluctuations in the CNY-Naira exchange rate have substantial impacts on Nigeria's trade dynamics. China's investment in infrastructure projects across Africa, including Nigeria, also links the Yuan directly to Nigeria's investment landscape (Adeniran, Popoola 2020). The South African Rand (ZAR) affects Nigeria's trade relations within Africa. As Nigeria's trade partner in sectors like energy and retail, the ZAR's fluctuations can influence the costs of cross-border trade, with potential effects on Nigeria's business environment and inflation rates (Olawale, Garwe 2021). Thus, these key global and BRICS currencies play significant roles in shaping Nigeria's economic interactions. From trade and investment flows to the management of Nigeria's foreign reserves, these currencies directly influence the Naira's exchange rate, inflation, and overall economic stability.

Empirical literature highlights these gaps, emphasizing the need for tailored methodologies that account for Nigeria's unique economic and policy environment. The estimation of VaR for the Nigerian Naira further underscores the complexities of managing financial risks where exchange rates can exhibit sudden and extreme

fluctuations. This challenge is particularly pronounced in Nigeria, where currency depreciation and speculative activities in the parallel market exacerbate volatility. The relevant research question is: What is the Value-at-Risk (VaR) for the Naira exchange rate in relation to the Brazilian Real, Russian Ruble, Indian Rupee, Chinese Yuan, and South African Rand using the VaR estimation method? Against the backdrop of the research question, the study seeks to forecast the exchange rate of the Naira and estimate its VaR in relation to emerging market currencies (Real, Rubles, Rupee, Yuan, Rand).

The significance of this research lies in its potential to advance knowledge in the fields of foreign exchange forecasting and risk estimation, particularly within the unique context of Nigeria's economy. As an emerging market, Nigeria faces persistent challenges in managing exchange rate volatility, which impacts macroeconomic stability and financial decision-making. Besides, foreign exchange forecasting and VaR analysis are pivotal in addressing the complexities of currency dynamics, particularly for the Nigerian Naira, which is prone to volatility due to factors such as oil price fluctuations, inflation, and macroeconomic instability. VaR, widely regarded as a cornerstone in risk management, quantifies the maximum potential loss of assets under adverse conditions within a defined time horizon. The V-C, MCS, and H-S add value by estimating various potential losses under conditions of uncertainty, making them particularly useful for addressing the risk of forex markets (Sun et al. 2019; Ren et al. 2021). Together, by evaluating risk assessment techniques like V-C, MCS, and H-S; these tools help mitigate the challenges posed by forex volatility, enabling stakeholders to navigate uncertainties more effectively.

The study contributes to existing literature by exploring the applicability of modern forecasting and risk estimation techniques in an emerging economy like Nigeria. Policymakers will benefit significantly from this research, as it provides empirical evidence on currency fluctuations and their effects on the Naira. Nigeria's dependence on oil exports, fluctuating foreign reserves, and exposure to external shocks make currency stability a crucial policy objective. By understanding the dynamics of how global currencies, including the USD, EUR, and CNY, interact with the Naira, policymakers can develop robust strategies to minimize economic vulnerabilities and ensure sustainable growth (Ibrahim et al. 2020).

The present research builds on this historical foundation to explore contemporary forecasting and risk management issues, bridging gaps in the empirical understanding of Nigeria's foreign exchange market (Ibekwe et al. 2022). This follows from the fact that the Nigeria's foreign exchange landscape has undergone significant shifts. From a fixed exchange rate regime in the 1960s to various forms of managed float systems, the Naira has experienced periods of stability and extreme volatility. Structural reforms in the 1980s, such as the Structural Adjustment Program (SAP), initiated the liberalization of the foreign exchange market, but the lack of consistent implementation has often led to instability. More recently, policy responses to external shocks, such as oil price declines and global financial crises, have further highlighted the challenges of managing exchange rate volatility in an open economy.

The research's contributions extend beyond its immediate findings. It addresses methodological gaps by exploring the suitability of advanced models like Monte Carlo (MCS) simulations and historical simulation (H-S) and the variance-Covariance (V-C) methods for evaluating risk of exchange rates of emerging market economies. This validates the effectiveness of these models and provides a framework for future studies in similar contexts, enhancing the academic discourse on financial risk estimation and exchange rate dynamics (Oyetade et al. 2019). This study is structured into five comprehensive sections to ensure a logical progression from the research problem to actionable insights. The first section introduces the study; it establishes the context and relevance of the study, particularly in the field of exchange rate forecasting and risk estimation for the Nigerian Naira. The second section provides a critical review of existing literature, focusing on exchange rate risk based on Valueat-Risk (VaR) estimation techniques. The third section details the research methodology, explaining the data collection, modeling, and analysis procedures. The fourth section presents the results of the analysis, offering a detailed interpretation of the findings. Section five concludes the research by highlighting some findings that contribute meaningfully to literature and practical applications while also emphasizing the study's limitations and propose directions for future research.

2. Previous scientific research findings

The VaR estimation has evolved to address exchange rate volatility challenges. Singh & Kumar (2024) employed the wavelet transform theory to estimate the VaR for the INR/USD exchange rate. Using daily exchange rate data from 2015 to 2023, the study demonstrated that wavelet-based models effectively captured multi-scale volatility dynamics, which are critical during rapid market shifts. The research highlighted the capability to identify localized spikes in volatility while maintaining accuracy over extended horizons. Johnson & Roberts (2024) applied the ensemble learning theory method to estimate the VaR for the GBP/USD exchange rate. The study demonstrated that ensemble methods, particularly boosting algorithms, excelled in improving predictive accuracy across various market conditions. The study emphasized that ensemble learning's adaptability to diverse conditions makes it essential for precise risk estimation. The study suggested that financial institutions adopt ensemble frameworks to enhance VaR accuracy and mitigate losses in dynamic currency markets.

Dlamini et al. (2024) combined GARCH and extreme value theory (EVT) to estimate the VaR for the ZAR/USD exchange rate, basing their approach in the hybrid model theory. Using daily exchange rate data from 2010 to 2023, the study demonstrated that the GARCH-EVT hybrid model provided superior estimates of extreme losses. This approach was particularly effective during high-volatility periods, accurately capturing rare but impactful events in the currency market. The findings stated the hybrid model's utility in managing currency risk, particularly for emerging markets with high exposure to external shocks.

Zhao et al. (2024) advanced Monte-Carlo historical simulation (MCS) by integrating agent-based models that capture behavioral and systemic feedback loops. For VaR estimation across market regimes, reliance on GARCH-family models dominates, with Ofori & Mensah (2023) highlighting their strengths during stable periods but limitations during crises. The authors applied the historical simulation theory to estimate the VaR for the NGN/USD exchange rate, emphasizing its reliance on past market behavior as a predictor of future risks. Using exchange rate data from 2015 to 2022, the study found that MCS produced reliable VaR estimates during

periods of market stability. It suggested integrating extreme value theory (EVT) with historical simulation to enhance its capability in modeling extreme tail risks. The study resolved that such hybrid approaches would improve VaR accuracy in highly volatile markets like Nigeria's. Nonetheless, its accuracy declined during crises when market dynamics significantly deviated from historical norms. Besides, their focus on developed economies limits applicability to Nigeria's diverse volatility drivers.

Ali et al. (2023) explored the use of machine learning algorithms, including decision trees and random forests, for VaR estimation of the PKR/USD exchange rate, basing their work on ensemble learning theory. Using data spanning 2010 to 2022, the study demonstrated that ensemble methods provided more accurate and stable risk estimates, particularly in volatile settings. The findings highlighted the robustness of ensemble approaches in adapting to diverse market conditions and managing uncertainty. López & Martinez (2022) relied on Quantile Regression (QR) method to estimate the VaR for the MXN/USD exchange rate, emphasizing QR's suitability for analyzing the tails of return distributions. The study utilized daily exchange rate data from 2015 to 2021, focusing on the lower quantiles to estimate downside risks during periods of market volatility. Results showed that QR outperformed traditional linear regression models by providing more accurate and tailored risk estimates under extreme conditions. The findings underlined QR's ability to adapt to market regimes and its theoretical relevance for asymmetric risk analysis. The resolved that incorporating QR into risk management practices could enhance the precision of VaR estimates, particularly for currencies prone to high volatility, such as the MXN/USD pair.

Müller et al. (2022) employed the Component GARCH (CGARCH) model, rooted in volatility decomposition theory, to estimate the VaR of the CHF/USD exchange rate. Using daily data spanning 2010 to 2020, the study revealed that the CGARCH model effectively captured persistent volatility trends and transitory fluctuations. The results showed that the model provided superior VaR estimates compared to standard GARCH models, particularly during periods of sustained volatility. It established that CGARCH aligns with the theoretical framework of volatility decomposition, offering a valuable approach for improving currency risk forecasts in stable and volatile market settings. Zhang et al. (2022), drawing from the support vector machine (SVM) theory, examined the integration of SVM with MCS (SVM-MCS) for estimating the VaR of the JPY/USD exchange rate. The SVM theory posits that the algorithm captures complex, non-linear relationships by finding optimal boundaries between data points in high-dimensional spaces. Utilizing exchange rate data from 2011 to 2021, the study demonstrated that the hybrid SVM-MCS approach outperformed standalone SVM and traditional parametric models in accurately estimating tail risks. The study endorsed adopting SVM-MCS frameworks for financial institutions to improve risk modeling and mitigate potential losses during volatile market phases.

Kim & Choi (2021) applied deep learning models rooted specifically long shortterm memory (LSTM) networks, to estimate the VaR for the USD/KRW exchange rate. The study utilized exchange rate data from 2010 to 2020, finding that LSTM models significantly outperformed traditional GARCH models in predictive accuracy and computational efficiency. The research established that LSTM models offer a superior approach for VaR estimation in volatile markets, recommending their adoption in modern financial risk assessment practices.

Silva et al. (2021) assessed the combination of GARCH and Artificial Neural Networks (ANN) for estimating the VaR of the BRL/USD exchange rate. Using data spanning 2010 to 2020, the study found that the hybrid GARCH-ANN model effectively captured non-linear volatility patterns and offered superior risk estimates compared to standalone GARCH models. The research highlighted that the hybrid model's ability to adapt to changing market dynamics and volatility made it an essential tool for financial risk management in emerging markets. Silva et al. suggested further exploration of hybrid frameworks to enhance VaR estimation accuracy in complex financial systems. Wang et al. (2020) utilized Copula theory to estimate VaR for the CNY/USD exchange rate, highlighting the theory's capability to model dependency structures between variables independent of their marginal distributions. Analyzing high-frequency data from 2010 to 2018, demonstrated that copula-based models effectively captured joint dependencies and tail behaviors, outperforming traditional approaches in volatile market conditions. The findings underscored the theoretical significance of copulas in addressing the complexities of exchange rate dynamics, particularly during periods of heightened uncertainty.

ANALYSIS OF VALUE-AT-RISK (VAR) OF NAIRA AGAINST BRICS CURRENCIES

The reviewed studies on exchange rate forecasting and VaR estimation reveal gaps remain that necessitate additional research, particularly in the context of emerging markets such as Nigeria. These include the integration of advanced forecasting methods, such as MCS, H-S, and V-C methods for robust VaR estimation of the Naira exchange rate in volatile markets. The MCS has shown success in forecasting exchange rate volatility in advanced and emerging economies, as demonstrated by Chen & Zhang (2019) and Williams et al. (2020). Nonetheless, its application to African currencies, particularly the Nigerian Naira (NGN), remains limited. Ahmed et al. (2021) and Okafor & Adeyemi (2022) explored Nigeria's context but lacked comprehensive modeling of external shocks and non-linearities, emphasizing short-term horizons. While Ahmed et al. (2021) applied MCS to the NGN/USD exchange rate considering oil price fluctuations; their focus on oil-related shocks overlooks other significant drivers, such as market sentiment and behavioral dynamics. Besides, the hybrid approaches utilized by Zhao et al. (2024) have not yet been extended to the Naira exchange rate.

Existing researches are less relevant to the specific obstacles faced by emerging economies since the majority of research focuses on advanced economies with reliable financial systems and easily accessible data. The evaluation of African currencies, such as the NGN, which is impacted by changes in the price of oil, trade imbalances, and political unrest, is seriously lacking. Nigeria's major trading currencies, including regional rivals and new partners like the CNY, are mostly ignored in research that focusses on international currency pairs like USD/EUR or USD/JPY. This omission restricts the findings' relevance to the economic environment of Nigeria. There is a dearth of thorough comparisons between the MCS and V-C methods of predicting the VaR of the Naira exchange rate. While evaluating the advantages and disadvantages of MCS and ARIMA, Okafor & Adeyemi (2022) neglected to take exogenous shocks or medium- to long-term forecasting into account. This present work provides a thorough analysis of these approaches and their consequences for estimating VaR in the economic setting of Nigeria. By measuring VaR using the MCS and H-C methodologies for currencies essential to Nigeria's trade and investment landscape, this study closes this gap. Furthermore, there is a lack of a comprehensive framework that integrates risk estimation and forecasting techniques; instead, research have addressed them separately (e.g., Ahmed et al. 2021; Chen et al. 2019).

Another population gap lies in the narrow temporal scope of many studies, which frequently exclude data from periods of economic instability, such as the Covid-19 pandemic and sectoral analyses of critical areas like agriculture and informal trade. These omissions limit understanding of how exchange rate volatility affects diverse economic actors. This study addresses these gaps by focusing on the Naira, key trading currencies, and a broader range of stakeholders, including SMEs and individuals. It also incorporates data from periods of instability and examines sectoral impacts to provide a more inclusive and context-specific analysis of exchange rate forecasting and risk estimation for Nigeria. By integrating advanced forecasting techniques, external variables, and systematic comparisons, this research aims to address the gaps identified and contribute meaningfully to the fields of exchange rate forecasting and risk management. Thus, this study addresses these gaps by applying MCS to forecast exchange rates of selected currencies against the Nigerian Naira, incorporating external variables such as interest rate differentials and inflation rate differentials. This study addresses these gaps by extending MCS and ARIMA applications to multiple currencies, incorporating advanced VaR estimation technique, and integrating forecasting and risk frameworks. This will provide actionable insights for policymakers and financial stakeholders managing Nigeria's volatile exchange rate environment.

3. Methodology

In foreign exchange markets, Modern Portfolio Theory (MPT) provides a framework for assessing risk-return trade-offs in currency investments. The MPT, developed by Markowitz (1952), emphasizes the importance of diversification in portfolio construction to optimize the Naira exchange rate returns against the currencies of the BRICS for a given level of risk. Besides, Obi et al. (2022) utilized MPT to optimize currency portfolios under volatile conditions. These unpredictable circumstances inform the methodological approach of this study, particularly the

integration of VaR models and MCS for risk assessment and forecasting. Since MPT provides a robust framework for assessing and quantifying risk associated with exchange rate movements through VaR, VaR analysis is useful for implementing risk management measures, such as adjusting currency positions or hedging instruments like options and forwards, to mitigate the risks of adverse currency movements. The VaR was estimated using the V-C method, sometimes referred to as the parametric method, which accounts for the exchange rate's sensitivity to changes in price. 5,110 days of historical data for the daily exchange rate returns and losses of the Naira and the currencies of the BRICS nations were used to estimate the V-C matrices displayed below.

$$Var - Cov = \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/BRL} \\ \sigma_{NGN/BRL} & \sigma_{NGN}^{2} \end{bmatrix}, Var - Cov = \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/RUB} \\ \sigma_{NGN/RUB} & \sigma_{NGN}^{2} \end{bmatrix}$$
$$Var - Cov = \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/INR} \\ \sigma_{NGN/INR} & \sigma_{NGN}^{2} \end{bmatrix}, Var - Cov = \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/CNY} \\ \sigma_{NGN/CNY} & \sigma_{NGN}^{2} \end{bmatrix}, Var - Cov = \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/CNY} \\ \sigma_{NGN/CNY} & \sigma_{NGN}^{2} \end{bmatrix}$$

The volatility of the aggregate changes in exchange rate losses was articulated as a function of the vector of market-price sensitivities, the standard deviations of the exchange rate returns between the Naira and the BRICS currency as well as the covariance between the two currencies. Thus, the standard deviation of changes in the exchange rate between the Naira and each of the BRICS currencies is given as:

$$\begin{split} S_{NGN/BRL} &= \sqrt{\begin{bmatrix} \delta_{NGN} & \delta_{BRL} \end{bmatrix}} \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/BRL} \\ \sigma_{NGN/BRL} & \sigma_{NGN}^{2} \end{bmatrix} \begin{bmatrix} \delta_{NGN} \\ \delta_{BRL} \end{bmatrix}}, \\ S_{NGN/INR} &= \sqrt{\begin{bmatrix} \delta_{NGN} & \delta_{INR} \end{bmatrix}} \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/INR} \\ \sigma_{NGN/INR} & \sigma_{NGN}^{2} \end{bmatrix} \begin{bmatrix} \delta_{NGN} \\ \delta_{INR} \end{bmatrix}}, \\ S_{NGN/INR} &= \sqrt{\begin{bmatrix} \delta_{NGN} & \delta_{INR} \end{bmatrix}} \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/INR} \\ \sigma_{NGN/INR} & \sigma_{NGN}^{2} \end{bmatrix} \begin{bmatrix} \delta_{NGN} \\ \delta_{INR} \end{bmatrix}}, \\ S_{NGN/INR} &= \sqrt{\begin{bmatrix} \delta_{NGN} & \delta_{INR} \end{bmatrix}} \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/INR} \\ \sigma_{NGN/INR} & \sigma_{NGN}^{2} \end{bmatrix} \begin{bmatrix} \delta_{NGN} \\ \delta_{INR} \end{bmatrix}}, \\ S_{NGN/INR} &= \sqrt{\begin{bmatrix} \delta_{NGN} & \delta_{INR} \end{bmatrix}} \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/INR} \\ \sigma_{NGN/INR} & \sigma_{NGN}^{2} \end{bmatrix} \begin{bmatrix} \delta_{NGN} \\ \delta_{INR} \end{bmatrix}} \\ \end{bmatrix} \\ s_{NGN/INR} &= \sqrt{\begin{bmatrix} \delta_{NGN} & \delta_{IAR} \end{bmatrix}} \begin{bmatrix} \sigma_{NGN}^{2} & \sigma_{NGN/IAR} \\ \sigma_{NGN/IAR} & \sigma_{NGN}^{2} \end{bmatrix} \begin{bmatrix} \delta_{NGN} \\ \delta_{IAR} \end{bmatrix}} \\ \end{bmatrix}$$

Under this methodology, VaR was estimated by calculating the mean return and standard deviation of returns. Using these parameters, the potential losses over a given time period was estimated. The estimation of the VaR for the exchange rate between the Naira and any of the BRICS currencies for a given threshold of confidence was such that the standard deviation was scaled by the standard normal threshold factor. Considering that financial-asset returns and, consequently, exchange rate returns and losses are normally distributed that financial-asset returns and hence, exchange rate returns and losses are normally distributed. The appropriate scaling factors for the 99% and 95% levels of confidence are 2.33 and 1.645. As a robustness check, the MCS and Historical-Simulation (H-S) methods were used in addition to the V-C otherwise parametric method. The MCS is an advanced method that simulates several possible future events by using samples at random from a probability distribution. The MCS technique generates VaR in the FX context by determining the worst loss in the distribution and producing a range of potential future currency price movements. MCS's adaptability makes it possible to incorporate non-linearities which make it suitable for portfolios with intricate structures.

The core theory of the MCS method was the replication of numerous potential future price fluctuations that would have an impact on the value of the Naira relative to the BRICS currencies. The bivariate t-distribution with a correlation coefficient of 0.7955 was used to extract the exchange-rate returns and losses for currency pairs. A t-distribution was chosen for being able to measure the fat-tails characteristic observed in the data. The MCS generated series of pseudo-random figures to simulate macroeconomic factors and the financial variables of the study. One hundred thousand (100,000) simulations of the currency exchange rates were performed. For every currency pair, the Naira exchange rate was recalculated. Following a revaluation of the Naira exchange rate for every currency pair of simulated exchange rates, the differences between the current and the revalued return/losses were calculated for the NGN/BRL, NGN/RUB, NGN/INR, NGN/CNY, and NGN/ZAR exchange rate. Simulation of the effects of variables and market events that were not observed during the historical era but have the potential to occur is made possible by the MCS. A unique VaR estimate was obtained by analyzing the changes in exchange rate values.

Also, using historical data on the daily exchange rate of the Naira versus the BRICS currencies, the historical-simulation (H-S) approach calculates the VaR estimate by first applying each of the 5,110 currency pairs of the daily exchange rate changes that would have occurred if the current currency exchange rate had remained constant over those 5,110 trading days. This is followed by a revaluation of the exchange rates using the previous exchange rates of each nation, and then sorting the 5110 changes (losses/returns) in the value of the Naira exchange rate in order of magnitude to arrive at an observed distribution of changes in the exchange rates of the

Naira and BRICS currencies. This is because the H-S does not offer an underlying statistical assumption for the distribution of exchange rate losses and returns. So, the VaR estimate equals the percentile linked to the threshold of confidence.

Accordingly, the VaR framework which provides information into the forecasting dynamics between exchange rate losses and the implications for risk management fits so nicely with the objectives of the study. The VaR is a crucial risk management tool that measures possible losses at different confidence levels, which is consistent with MPT theories. This study estimates the VaR associated with the Naira exchange rate vis-à-vis the BRICS currencies by calculating the VaR for the chosen currency pairs. The MPT strengthens estimation of risk or losses particularly in dynamic markets. The VaR is a vital instrument in financial risk management since it measures the possible loss of the value of an asset or portfolio over a certain time period at a specific confidence level.

In foreign exchange (FX), VaR measures the greatest possible loss in a currency portfolio by taking into account the likelihood of unfavourable market changes over a predetermined period of time. The value of the VaR resides in its capacity to give financial organizations a tangible indicator of risk, allowing them to evaluate probable losses from exchange rate swings, allocate capital efficiently, and maintain adequate reserves to cover future losses. Regulators also utilize VaR to assess the solvency of financial organizations, establishing capital adequacy standards based on anticipated exchange rate losses (Bank for International Settlements 2022). By fusing the techniques of V-C, the MCS and H-S methods with the GJR-GARCH regression model, the VaR approach becomes a powerful tool for evaluating currency exchange risk, allowing analysts to quantify the potential losses associated with exchange rate fluctuations based on historical data and economic factors. Moreover, by analyzing how macroeconomic and financial factors influence exchange rate changes, this approach offers a valuable framework for risk management in currency portfolios. This helps to guarantee that possible losses remain within manageable bounds. The regression model can be expressed as follows:

 $EXCRAT = \alpha + \beta_1(m2/gdp) + \beta_2(Opr) + \beta_3(Ogr) + \beta_4(ERev) + \beta_5(Mpor) + \beta_6(Tbal) + \epsilon_t$ (1)

Where EXCRAT is the exchange rate (dependent variable) for the selected currency pairs (NGN/BRL, NGN/RUB, NGN/INR, NGN/CNY, NGN/ZAR), financial healthiness measured as changes in broad money supply as a percentage of output (m2/gdp), trade balance-deficit or surplus (Tbal), external reserves (ERev), monetary policy rate (Mpor), changes in BRENT oil prices (Opr), output growth rate (OgR), α is constant term, β_1 , β_2 , β_3 , β_4 , β_5 , β_6 are coefficients measuring the impact of each independent variable on the exchange rate of the Naira in relation to each of the BRICS currencies, ε is error term capturing unobserved factors affecting the exchange rate. The model is further enhanced using advanced econometric techniques to ensure robustness and predictive accuracy. The model offers an additional layer of risk quantification by incorporating volatility measures and macroeconomic indicators into the VaR estimation, making it a more comprehensive tool for risk management.

VaR is frequently used in the management of foreign exchange portfolios of currencies since it aids in evaluating and controlling the risks related to changes in currency prices. VaR can be used to calculate the overall risk of a multi-currency portfolio by adding the exchange rate risk of each individual currency holding. This makes it possible for financial firms to establish risk thresholds, distribute funds efficiently, and create hedging plans to reduce any losses. The study examines the exchange rate changes of the Naira in relation to five emerging market currencies: the South African Rand, the Indian Rupee, the Chinese Yuan, the Russian Ruble, and the Brazilian Real. A number of important international and emerging market currencies have an impact on the Nigerian economy, which is reflected in the Nigerian Naira (NGN), the country's currency. A specific time frame, from January 1, 2010 to December 31, 2024, was covered for the analysis, which makes use of daily exchange rate data. The m2/gdp ratio was calculated by adding up the currency outside the banking system, demand deposits, time, savings, and foreign currency deposits, as well as bank and traveler's checks and other securities like commercial paper. The m2/gdp ratio measures the healthiness of the financial industry of the countries in the study. Then sources of these data were the databases of the World Bank (https://data.worldbank.org/indicator).

4. Results

This section systematically addresses the study objectives by presenting the empirical findings and providing an in-depth discussion of the implications for services trade policies in the examined regions. Results are analyzed in light of theoretical expectations and prior empirical studies to draw meaningful conclusions. All of the variables are integrated of order I(1), according to the results in the Appendix; the results of the co-integration test for the Russian Ruble (RUB) vs the Nigerian Naira (NGN) point to an extended equilibrium connection between the independent variables and the exchange rate (EXCRAT). The VaR of NGN/BRL exchange rate based on V-C in Table 1 provides insights into potential risks associated with exchange rate fluctuations. In this model, the mean return (μ) is 2.77050, which represents the expected return based on historical data. The Z-value $(Z_{0.05})$ and Zvalue (Z_{0.01}) are 1.645 and 2.33 corresponding to 95% and 99% confidence levels, indicating the likelihood that the losses will not exceed the estimated VaR. The conditional variance (σ^2) of 308.64587 is calculated using a V-C model, which accounts for time-varying volatility, while the conditional standard deviation (σ) is 17.56832. The VaR at the 95% and 99% confidence level are 28.89989, and 40.93419 giving the maximum expected losses over one day, with 5% and 1% probabilities that losses will exceed the VaR values.

Table 2 summarizes the VaR of NGN/BRL exchange rate based on H-S while Table 3 presents the VaR results of NGN/BRL exchange rate using the MCS. The MCS VaR estimates of NGN/BRL exchange rate at the 95% and 99% threshold levels include 126.1792 and 425.3150; while the H-S method, the VaR estimates for the 95% and 99% levels of thresholds are 26.34855 and 37.33921 respectively.

Parameter	Value	Narration
Mean Return (μ)	2.77050	The constant term from the mean equation represents the expected return
Z-Value (Z _{0.05})	1.645	Z-score for a 95% confidence level
Z-Value (Z _{0.01})	2.33	Z-score for a 99% confidence level
Conditional Variance (σ2)	308.6587	Variance estimate
Conditional Std. Dev. (o)	17.5632	Standard deviation, $\sigma 2$
VaR (VaR _{95%})	28.89989	Maximum expected loss at 95% confidence for one day
VaR (VaR99%)	40.93419	Maximum expected loss at 99% confidence for one day
Confidence Level	95%	Indicates that losses will not exceed the VaR in 95% of the cases
Confidence Level	99%	Indicates that losses will not exceed the VaR in 99% of the cases
Time Frame	Daily	Each VaR result applies to a single day from January 1, 2010 – December 31, 2024)

Table 1. Value-at-Risk (VaR) of NGN/BRL exchange rate based on variance-covariance

Source: authors' estimation results (2025).

 Table 2. Value-at-Risk (VaR) of NGN/BRL exchange rate based on historical simulation

Parameter	Value	Narration
VaR (VaR _{95%})	26.34855	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	37.33921	Maximum expected losses at 99% confidence for one day

ANALYSIS OF VALUE-AT-RISK (VAR) OF NAIRA AGAINST BRICS CURRENCIES

Table 3: Value-at-Riks (VaR) of NGN/BRL exchange rate based on Monte-Carlo simulation

Parameter	Value	Narration
VaR (VaR _{95%})	126.1792	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	425.3150	Maximum expected losses at 99% confidence for one day

Source: authors' estimation results (2025).

Table 4. Value-at-Risk (VaR) of NGN/RUB exchange rate based on variance-covariance

Parameter	Value	Narration
Mean Return (µ)	60.94285	The constant term from the mean equation represents the expected return.
Z-Value $(Z_{0.05})$	1.645	Z-score for a 95% confidence level
Z-Value (Z0.01)	2.330	Z-score for a 99% confidence level
Conditional Variance (o2)	434.9352	Variance estimate
Conditional Std. Dev. (o)	20.8551	Standard deviation, $\sigma 2$
VaR (VaR95%)	34.30664	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	48.59238	Maximum expected losses at 99% confidence for one day
Confidence Level	95%	Indicates that losses will not exceed the VaR in 95% of the cases
Confidence Level	99%	Indicates that losses will not exceed the VaR in 99% of the cases
Time Frame	Daily	Each VaR result applies to a single day from January 1, 2010 – December 31, 2024).

The potential risks in exchange rate changes between RUB and NGN are evaluated by the VaR of the NGN/RUB exchange rate using the V-C approach. The estimates are shown in Table 4. The expected return is indicated by the mean return (μ), which is 60.94285. With 95% and 99% confidence thresholds, respectively, the Z_{0.05} and Z_{0.01} are 1.645 and 2.330, indicating 5% and 1% likelihood that losses will surpass the estimated VaR. In order to account for time-varying volatility, the V-C model yields a conditional variance (σ^2) of 434.9352, and the conditional standard deviation (σ) of 20.8551, which shows the degree of variation in returns. The VaR, which represents the maximum projected losses per day with 95% and 99% probabilities that losses will not surpass these values, are 34.30664 and 48.59238, respectively.

Table 5. Value-at-Risk of NGN/RUB exchange rate based on historical simulation

Parameter	Value	Narration
VaR (VaR _{95%})	30.18745	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	42.19378	Maximum expected losses at 99% confidence for one day

Source: authors' estimation results (2025).

Table 6 presents the VaR estimates of the NGN/RUB exchange rate using the H-S method for both the 95% and 99% threshold of confidence. These estimates— 30.18745 and 42.19378, respectively—compare closely with the estimates derived from the V-C method. Table 6 displays the VaR of the NGN/RUB exchange rate using the MCS, which are comparatively larger, at 192.0134 and 317.1595, respectively.

 Table 6: Value-at-Risk (VaR) of NGN/RUB exchange rate based on Monte-Carlo simulation

Parameter	Value	Narration
VaR (VaR _{95%})	192.0134	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	317.1595	Maximum expected losses at 99% confidence for one day

Table 7 shows that the projected mean return (μ) of 57.36302. This serves as a baseline for future forecasts using the Indian Rupee's exchange rate against the Nigerian Naira. For 95% and 99% confidence levels, the Z-values of 1.645 and 2.330 represent the critical value, indicating that the odds of losses surpassing the computed VaR are just 5% and 1%, respectively. To account for variabilities, the V-C model was used to calculate the conditional variance (σ^2), which came out to be 228.87454. The average fluctuation in returns is measured by the conditional standard deviation (σ), which is 0.9186 and is calculated as the square root of variance. The highest projected losses per day are represented by the estimated VaR at 95% confidence, which is 24.88655 and 35.24964, guaranteeing that these losses only happen by 5% and 1%.

 Table 7. Value-at-Riks (VaR) of NGN/INR exchange rate based on variancecovariance

Parameter	Value	Narration
	57.2(202	The constant term from the
Mean Return (µ)	57.36302	the even estad native
		the expected return.
Z-Value $(Z_{0.05})$	1.645	Z-score for a 95%
(=0.03)		confidence level
7 Value (7)	2 3 3 0	Z-score for a 99%
Σ -value ($\Sigma_{0.01}$)	2.550	confidence level
	229.97454	Variance from Variance-
Conditional Variance ($\sigma 2$)	228.87454	Covariance model
		Standard deviation, derived
Conditional Std. Dev. (σ)	15.1286	$as \sigma^2$
		Maximum expected losses
VaR (VaRosov)	24 88655	at 95% confidence for one
vure (vuregs/6)	24.00055	day
		Maximum avposted losses
$\mathbf{V}_{\mathbf{P}}\mathbf{D}$ ($\mathbf{V}_{\mathbf{P}}\mathbf{D}$)	25 24064	at 00% confidence for one
vak (vak _{99%})	35.24964	at 99% confidence for one
		day
		Indicates that losses will not
Confidence Level	95%	exceed the VaR in 95% of
		the cases
		Each VaR result applies to a
Time Frame	Daily	single day from January 1,
		2010 – December 31,
		2024).

Parameter	Value	Narration
VaR (VaR _{95%})	19.28794	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	31.29475	Maximum expected losses at 99% confidence for one day

Table 8. Value-at-Risk of NGN/INR exchange rate based on historical simulation

Source: authors' estimation results (2025).

Table 8 shows the VaR estimates of the NGN/INR exchange rate for both the 95% and 99% threshold of confidence using the H-S method. These estimates 19.28794 and 31.29475 respectively are closely related in magnitude with the VaR estimates computed with the V-C method. On the contrary, the VaR of NGN/INR exchange rate given by 113.4809 and 225.3975 for the 95% and 99% threshold of confidence using the MCS are relatively larger. These are reported in Table 9 below.

 Table 9. Value-at-Risk (VaR) of NGN/INR exchange rate based on Monte-Carlo simulation

Parameter	Value	Narration
VaR (VaR95%)	113.4809	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	225.3975	Maximum expected losses at 99% confidence for one day

Source: authors' estimation results (2025).

The VaR estimates of the NGN to CNY exchange rate are shown in Table 10 from the V-C method. The projected average return is represented by the mean return (μ) of 5.565377. The 95% and 99% confidence thresholds are represented by the Z-values (Z_{0.05}) of 1.645 and Z-values (Z_{0.01}) of 2.33, respectively, which indicate that the likelihood of losses exceeding the estimated VaR is only 5% and 1%. Time-varying volatility is taken into account by the conditional variance (σ^2) of 100.79457, and the degree of return fluctuation is reflected by the conditional standard deviation (σ), which is 10.03965. In terms of 95% and 99% probability that losses will not surpass

ANALYSIS OF VALUE-AT-RISK (VAR) OF NAIRA AGAINST BRICS CURRENCIES

these values, the VaR at 95% and 99% confidence thresholds are 16.51522 and 23.39238, which represent the maximum expected loss for a day.

Parameter	Value	Narration
		The constant term from the
Mean Return (µ)	5.565377	mean equation represents
		the expected return.
Z Value (Z)	1 645	Z-score for a 95%
Σ -value ($\Sigma_{0.05}$)	1.045	confidence level.
Z Value (Z)	2 220	Z-score for a 99%
Σ -value ($\Sigma_{0.05}$)	2.330	confidence level.
		Variance computed using
Conditional Variance (σ 2)	100.79457	the Variance-Covariance
		model
Conditional Std. Dev. (σ)	10.03965	Standard deviation, $\sigma 2$
	1(51522	Maximum expected loss at
Var (Var _{95%)}	10.31322	95% confidence for one day
	22 20220	Maximum expected loss at
Vak (Vak99%)	23.39238	95% confidence for one day
		Indicates that losses will not
Confidence Level	95%	exceed the VaR in 95% of
		the cases.
		Indicates that losses will not
Confidence Level	99%	exceed the VaR in 99% of
		the cases.
		Each VaR result applies to a
Time Frame	Daily	single day from January 1,
	-	2010 – December 31, 2024)

 Table 10. Value-at-Risk (VaR) of NGN/CNY exchange rate based on variancecovariance

Source: authors' estimation results (2025).

Table 11. Value-at-Risk (VaR) of NGN/CNY exchange rate based on historical simulation

Parameter	Value	Narration
VaR (VaR _{95%})	13.49751	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	22.10389	Maximum expected losses at 99% confidence for one day

As shown in Table 11, the VaR estimates of the NGN/CNY exchange rate for both the 95% and 99% threshold of confidence using the H-S method are 13.49751 and 22.10389 respectively as against 106.8091 and 314.1256 estimated with the MCS method. A critical examination of these VaR estimates shows that the VaR of NGN/CNY exchange rate given by MCS as presented in Table 12 are relatively larger.

 Table 12. Value-at-Risk (VaR) f NGN/CNY exchange rate based on Monte-Carlo simulation

Parameter	Value	Narration
VaR (VaR95%)	106.8091	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	314.1256	Maximum expected losses at 99% confidence for one day

Source: authors' estimation results (2025).

Table 13 presents the VaR estimates of the NGN/ZAR exchange rate derived from the V-C method. The predicted return for the exchange rate is 9.35823, and the thresholds for substantial deviations are shown by the Z-values of 1.645 and 2.330, and both defines the 95% and 99% confidence levels, respectively. The exchange rate return variability is represented by the conditional variance of 338.54197. The exchange rate risks are represented by the conditional standard deviation of 18.39951. The VaR estimates, which represent the entire projected loss for a single day, are 30.26719 and 42.87086 at 95% and 99% confidence thresholds respectively. These indicate that the NGN/ZAR currency rate will not drop by more than N30.3 and N42.9 during the day, with 95% and 99% confidence levels.

Parameter	Value	Narration
Mean Return (µ)	9.35823	The constant term from the mean equation represents the expected return
Z-Value (Z _{0.05})	1.645	Z-score for a 95% confidence level
Z-Value (Z _{0.01})	2.330	Z-score for a 99% confidence level
Conditional Variance (o2)	338.54197	Variance estimate
Conditional Std. Dev. (σ)	18.39951	Standard deviation, σ^2
VaR (VaR95%)	30.26719	Maximum expected loss at 95% confidence for one day
VaR (VaR99%)	42.87086	Maximum expected loss at 99% confidence for one day
Confidence Level	95%	Indicates that losses will not exceed the VaR in 95% of the cases
Confidence Level	99%	Indicates that losses will not exceed the VaR in 99% of the cases
Time Frame	Daily	Each VaR result applies to a single day from January 1, 2010 – December 31, 2024)

 Table 13.
 Value-at-Risk (VaR)
 NGN/ZAR exchange rate based on variancecovariance

Source: authors' estimation results (2025).

 Table 14. Value-at-Risk (VaR) of NGN/ZAR exchange rate based on historicalsimulation

Parameter	Value	Narration
VaR (VaR _{95%})	25.38914	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	35.29847	Maximum expected losses at 99% confidence for one day

As shown in Table 14, the VaR estimates of the NGN/ZAR exchange rate for both the 95% and 99% threshold of confidence using the H-S method are 25.38914 and 35.29847 respectively as against 137.1293 and 339.5163 estimated with the MCS method. A statistical comparison of these VaR estimates shows that the VaR of NGN/ZAR exchange rate given by MCS as presented in Table 15 are relatively larger while those of H-S are strictly related in size with the estimates obtained from the V-C method.

 Table 15. Value-at-Risk (VaR) of NGN/ZAR exchange rate based on Monte-Carlo simulation

Parameter	Value	Narration
VaR (VaR _{95%})	137.1293	Maximum expected losses at 95% confidence for one day
VaR (VaR99%)	339.5163	Maximum expected losses at 99% confidence for one day

Source: authors' estimation results (2025).

Graphical plots of forecast results are crucial for evaluating the accuracy and reliability of predictive models. These plots compare actual values with forecasted ones, offering insights into how well the model captures trends and patterns. The time series forecast plot displays the actual and predicted values with confidence intervals, illustrating the uncertainty around predictions. Residual plots show the differences between actual and forecasted values, with random distribution around zero indicating a well-fitted model. A cumulative forecast error plot highlights potential biases, while forecast error density plots help assess whether errors follow a normal distribution. Close alignment between actual and predicted values, These visual analyses enhance the understanding of model performance and provide a foundation for improving predictive accuracy where necessary.

The NGN/BRL exchange rate's actual and predicted values, including ± 2 standard error (S.E.) bands, are displayed in the top graph of Figure 1. Whereas the dashed red lines show uncertainty, the blue line shows the forecast. The exchange rate fluctuated within the confidence interval but stayed comparatively constant between 2008 and

2018. The period from 2019 to 2022 had a substantial rise in volatility, and this is when the exchange rate peaked. Although rates are still high in comparison to previous years, the prognosis indicates stabilization starting in late 2022. Predictions for the NGN/BRL exchange rate show a decreasing uncertainty range, suggesting less volatility. The bottom graph shows the exchange rate variation, which increased after 2018 due to either policy changes or increased economic volatility. After 2022, there appears to be less uncertainty in future rates, according to the variance trends.

Model accuracy is reasonable, as confirmed by the RMSE of 1.28 and MAE of 0.93. A moderate level of predictive performance is shown by Theil's IC of 0.197. The model appears to be able to represent dynamic patterns well, that is, capable of accurately capturing dynamic patterns. As indicated by the variance (16.37%) and covariance proportion (72.77%). Although forecast trends indicate stabilization, they also emphasize the necessity of aggressive economic actions to maintain stability. According to the findings, it is important to keep an eye on changes in the money supply, external debt, and oil prices.





According to Figure 2, the forecast indicates gradual growth in the NGN/RUB exchange rate over time, with occasional fluctuations but staying within the confidence intervals. The exchange rate showed moderate variability between 2008 and 2018, followed by more consistent trends from 2020 onwards. The bottom graph, depicting forecast variance, shows a steep decline in uncertainty from 2008 to 2015, after which volatility remained minimal. The RMSE of 19.41 and MAE of 14.73 reflect the model's predictive performance, with a moderate MAPE of 43.80%. Theil's IC of 0.163 suggests reasonable forecasting accuracy. The variance proportion of 41.49% and bias proportion given by 31.90% indicate a balanced contribution of factors, while the covariance proportion (26.61%) highlights effective dynamic modeling. The reduced forecast variance implies growing confidence in predicting NGN/RUB exchange rates, emphasizing the importance of observing key drivers like changes in output growth, BRENT crude oil prices and external debt levels.



Figure 2. Graphical plots for NGN/RUB exchange rate

Source: authors' estimation results (2025).

The forecast results as reported in Figure 3 reveals a steady upward trend in the NGN/INR exchange rate over time, particularly from 2014 to 2022, with values mostly staying within the confidence bounds. The exchange rate displayed significant variability before 2015 but stabilized in later years. Forecasts indicate continued growth with narrowing uncertainty, reflecting greater stability. The bottom graph, which highlights forecast variance, shows high volatility between 2008 and 2012, declining sharply afterward and remaining minimal from 2015 onward. The RMSE of 11.03 and MAE of 7.41 demonstrate the model's strong predictive performance, with a moderate MAPE of 14.82%. Theil's IC of 0.082 signals high forecasting accuracy. Considering the variance and covariance proportions of 28.59% and 32.31% respectively; there is evidence to show that the model captures dynamics in NGN/INR effectively, while the bias proportion (39.09%) suggests some forecasting lag. Consistent trends and a lower forecast variance point to increased NGN/INR exchange rate predictability. Observing indicators such as trade balance, monetary policy rate, and BRENT crude oil prices remains vital for NGN/INR exchange rate management.



Figure 3. Graphical plots for NGN/INR exchange rate

Figure 4 shows the graphical analysis of the NGN/CNY exchange rate. The upper plot illustrates the actual and forecasted exchange rate (blue line) with ±2 standard error bounds (red dashed lines). The exchange rate shows a stable trend from 2008 to 2023, with minor fluctuations and a slight upward movement in recent years, indicating a potential mild depreciation of the Yuan against the Naira. Accuracy metrics reveal strong model performance, with a RMSE of 0.21, MAE of 0.17, and a low MAPE of 2.55%. The Theil IC of 0.02 suggests minimal forecasting errors, with most of the error attributed to the covariance proportion (0.89), reflecting a strong alignment between actual and forecasted values. The variance plot shows stability in the forecast except for a temporary spike around 2018, indicating unusual volatility during that period. According to the estimate, the Yuan would continue to move gradually in relation to the Naira, impacted by the interaction of economic variables including inflation and interest rates. When making decisions on trade and investment using these currencies, these findings are essential.





Source: authors' estimation results (2025).

ANALYSIS OF VALUE-AT-RISK (VAR) OF NAIRA AGAINST BRICS CURRENCIES

The topmost plot for the NGN/ZAR exchange rate as reported in Figure 5 shows plot shows the actual and forecasted exchange rate as a blue line, while the red dashed lines represent the ± 2 standard error bounds, indicating forecast uncertainty. A gradual upward trend from 2008 to 2023 suggests consistent depreciation of the Rand against the Naira over the forecast period. The model's accuracy is indicated by metrics such as RMSE of 3.46, MAE of 2.29, and MAPE of 27.24%, which suggest moderate reliability. The Theil IC of 0.13 signifies a relatively good fit, while the bias proportion (0.32), variance proportion (0.23), and covariance proportion (0.45) explain the forecasting errors. The bottom plot highlights a declining forecast variance over time, signifying improved stability from 2010 onward. These results point to a continued depreciation of the ZAR against the NGN, influenced by the interplay of economic factors. This forecast offers valuable insights for understanding future exchange rate movements and potential financial implications.





Equation Type: Mean						
Variable Coefficient P-Value						
NGN/BRL(-1)	0.15429	0.0000				
Dgdpr	-0.47293	0.0020				
OPr	-0.21936	0.0000				
m2/gdp	1.92653	0.0001				
Ogr	0.14793	0.1567				
Tbal	-0.30982	0.1945				
ERev	0.28919	0.1789				
Mpor	-0.54911	0.0457				
С	1.17505	0.0011				
	Equation Type: Variance					
Variable	Coefficient	P-Value				
С	0.75722	0.0033				
ARCH(-1) ² , α	-0.15598	0.0000				
RESID(-1) ² *(RESID(-1)<0), γ	0.17290	0.0000				
GARCH(-1), eta	0.69376	0.0000				
R-squared = 0.85621: Adjusted R -squared 0.73289						

Table 16. GJR-GARCH results of NGN/BRL exchange rate

Source: authors' estimation results (2025).

Table 17. GJR-GARCH results of NGN/RUB exchange rate

Equation Type: Mean				
Variable	Coefficient	p-value		
NGN/RUB(-1)	-0.26439	0.0000		
Dgdpr	-0.17956	0.0000		
OPr	-0.32750	0.0245		
m2/gdp	1.03792	0.2651		
Ogr	-0.14623	0.0000		
Tbal	-0.15479	0.2571		
Erev	0.23879	0.2193		
Mpor	0.42157	0.0184		
С	0.94285	0.0000		
Equation Type: Variance				
Variable	Coefficient	P-value		
С	0.25791	0.2921		
ARCH(-1)^2, <i>α</i>	-0.13495	0.0001		
RESID(-1) ² *(RESID(-1)≤0), γ	0.18562	0.0000		
GARCH(-1), β	0.89256	0.0000		
R-squared =0.78132; Adjusted R-squared = 0.61375				

Equation Type: Mean						
Variable Coefficient P-value						
NGN/INR(-1)	-0.41879	0.1161				
Dgdpr	-0.25763	0.0004				
OPr	-1.15042	0.0000				
m2/gdp	0.16728	0.1672				
Ogr	0.03576	0.2389				
Tbal	-1.02798	0.0000				
ERev	0.14735	0.2861				
Mpor	-0.18725	0.0000				
С	5.16302	0.0000				
E	Equation Type: Variance					
Variable	Coefficient	P-value				
С	0.72355	0.2921				
ARCH(-1)^2, α	-0.10932	0.0001				
RESID(-1) ² *(RESID(-1)<0), γ	0.14246	0.0000				
GARCH(-1), β	0.66307	0.0052				
$R_{-squared} = 0.657114$ · A divised $R_{-squared} = 0.509970$						

Table 18. GJR-GARCH results of NGN/INR exchange rate

Source: authors' estimation results (2025).

Table 19. GJR-GARCH results of NGN/CNY exchange rate

Equation Type: Mean				
Variable	Coefficient	P-value		
NGN/CNY(-1)	0.63215	0.0093		
Dgdpr	-0.12691	0.0006		
OPr	-0.34562	0.0022		
m2/gdp	1.0387	0.1467		
Ogr	0.102861	0.0000		
Tbal	0.18735	0.1672		
ERev	0.13910	0.0001		
Mpor	-0.25617	0.0000		
С	5.565377	0.0000		
Equation Type: Variance				
Variable	Coefficient	P-value		
С	0.23597	0.3243		
ARCH(-1)^2, α	0.14623	0.0000		
RESID(-1) ² *(RESID(-1)<0), γ	0.15294	0.0000		
GARCH(-1), eta	0.76315	0.0006		
R-squared = 0.58931: Adjusted R-squared 0.509970				

Equation Type: Mean				
Variable	Coefficient	P-value		
NGN/ZAR(-1)	0.31986	0.0000		
Dgdpr	-0.11256	0.0000		
OPr	0.07834	0.0000		
m2/gdp	0.15612	0.0000		
Ogr	-0.14520	0.1863		
Tbal	-0.01892	0.2579		
ERev	1.03265	0.0000		
Mpor	-0.01893	0.1772		
С	1.35250	0.0000		
Equation Type: Variance				
С	0.17426	0.0040		
ARCH(-1)^2, α	0.11519	0.0141		
RESID(-1) ² *(RESID(-1)<0),				
γ	0.13346	0.0000		
GARCH(-1), β	0.59123	0.0001		
R-squared = 0.82371. Adjusted R -squared = 0.791544				

Table 20. GJR-GARCH results of NGN/ZAR exchange rate

Source: authors' estimation results (2025).

5. Discussion

Results from an evaluation of the three VaR estimation techniques vary greatly. The strategy that produces the lowest risk estimations is the H-S method, which considers the actual shape of the observed distribution of losses and returns. According to the H-S technique, the VaR estimates of 26.34855, 30.18745, 19.28794, 13.49751, and 25.38914, respectively, correspond to the fifth percentile of the distribution of changes in the exchange rate values (profit or losses) of NGN/BRL, NGN/RUB, NGN/INR, NGN/CNY, and NGN/ZAR at a 95% threshold level. Relatively, for a 99% level of threshold the VaR estimates are 37.33921; 42.19378; 31.29475; 22.10389; and 35.29847 all equals the first percentile of the NGN/BRL, NGN/RUB, NGN/INR, NGN/CNY, NGN/ZAR exchange rate returns/losses.

ANALYSIS OF VALUE-AT-RISK (VAR) OF NAIRA AGAINST BRICS CURRENCIES

The NGN/BRL, NGN/RUB, NGN/INR, NGN/CNY, and NGN/ZAR exchange rates have MCS VaR estimates of 126.1792 and 425.3150, 192.0134 and 317.1595, 113.4809 and 225.3975, 106.8091 and 314.1256, and 137.1293 and 339.5163 at the 95% and 99% threshold levels, respectively. Earnings and losses are given the same weight in the V-C estimates, which produces VaR estimates that are only slightly consistent with the VaR estimates of the H-S approach. This does, in fact, establish how stable the VaR of the Naira exchange rates is in respect to the BRICS countries' currencies. The MCS VaR estimates were significantly higher than the VaR estimates provided by the V-C and H-S methods. The probability of tail events is the main focus of a VaR model, the t-distribution upon which the MCS simulation was done, with a few exceptions, showed longer tails than the normal distribution, which has a higher proportion of its probability mass in the distribution's tails. Hence, the VaR estimates emanating from the MCS became much larger than those given by V-C and H-S methods. Noticeably, this effect becomes remarkable (the larger the VaR estimate) the higher the threshold of confidence.

The V-C VaR estimates of the Naira's exchange rate against the BRICS currencies are 28.89989 and 40.93419; 34.30664 and 48.59238; 24.88655 and 35.24964; 16.51522 and 23.39238; and 30.26719 and 42.87086 for the NGN/BRL, NGN/RUB, NGN/INR, NGN/CNY, and NGN/ZAR values, at the 95% and 99% threshold levels respectively. The VaR estimate clearly increases with the degree of confidence. There is a 5% chance that the NGN/BRL currency rate will lose more than 28.89989 Naira, according to statistical inference, and a 1% chance that the NGN/BRL exchange rate will lose more than 40.93419 Naira. Comparatively, the estimates show that the NGN/RUB currency rate will lose more than 34.30664 Naira at a 5% chance level and more than 48.59238 Naira at a 1% probability level. Additionally, there is a 5% chance that the NGN/INR currency rate will lose more than 24.88655 Naira, and a 1% chance that the NGN/RUB exchange rate will lose more than 35.24964 Naira. The likelihood that the NGN/CNY exchange rate will lose more than 16.51522 Naira is 5%, and at the 1% probability threshold, the loss will exceed 23.39238 Naira. There is a 5% probability that the loss on the NGN/ZAR currency exchange rate is above 30.26719 Naira but at 1% probability, the loss on the NGN/ZAR will be larger than 42.87086 Naira.

The study's results agree with those of Ahmed et al. (2021), and Gupta & Patel (2020). Ahmed et al. (2021) used extreme value theory (EVT) to estimate the VaR of the NGN/USD exchange rate, highlighting EVT's theoretical focus on modeling the tails of return distributions where extreme losses occur. By examining exchange rate data from 2012 to 2020, Ahmed et al. (2021) showed that EVT is superior in identifying extreme market risks, particularly during financial crises when traditional models tend to underestimate risks. The study concluded that EVT provides a crucial tool for precise risk estimation in extremely volatile markets and suggested incorporating it into risk management frameworks for emerging economies like Nigeria. Gupta & Patel (2020) investigated the VaR estimation for the INR/USD exchange rate, building on the MCS theory as a foundation. The study simulated exchange rate movements using historical data from 2014 to 2019 and estimated VaR under various market conditions. The results showed that, in comparison to conventional parametric techniques, MCS provided more accurate risk estimations during extreme market occurrences and successfully captured tail risks. In stresstesting scenarios, which are crucial for emerging markets with volatile currencies, the study underlined that MCS's ability to model non-normal distributions offered a notable benefit. It emphasized MCS's resilience and flexibility in unpredictable economic conditions and pushed for its inclusion in risk management frameworks for financial firms and central banks.

Table 16 presents the findings of the conditional volatility of the BRL/NGN exchange rate. The estimations shed light on the variables that influence exchange rate volatility. With a coefficient of -0.15598 and a probability value of 0.0000, the ARCH(-1)^2 term, which represents lagged squared residuals, shows that historical shocks (residuals) have a substantial impact on the current volatility of the NGN/BRL exchange rate. This finding implies that recent disruptions significantly influence how volatile the currency pair is. The GJR-GARCH(1,1) model's persistence in volatility was determined by -0.15598 + 0.17290 + 0.69376/2 = 0.35534 < 1. For a leverage effect, we examined the sign, magnitude, and statistical significance of γ . The sign of $\gamma > 0$ (0.17290> 0). The magnitude of $\gamma | 0.17290 |$ is positive and the zero p-value indicates significance. fulfilling the requirements for non-negativity of variance equation coefficients, 0.75722 > 0; $\alpha > 0$ (i.e., coefficient of RESID(-1)^2); $\beta >= 0$

(i.e., coefficient of GARCH(-1)); $\alpha + \gamma \ge 0$. In other words, the variance equation results are valid as long as -0.15598 + 0.17290 = 0.01692 >= 0. The findings show that volatility persists. This suggests that times of high volatility may not always be followed by periods of lower volatility to reflect a natural adjustment process in the foreign currency market, and it does not imply mean-reverting behaviour in the NGN/BRL exchange rate volatility. Also, the model's adjusted R-squared value of 0.73289 indicates 73% of the variation in the NGN/BRL exchange rate is explained by the conditional variations of the model as well as the fluctuations in the independent variables.

The conditional volatility results of the NGN/RUB exchange rate are reported in Table 17. The ARCH(-1)² term, which represents lagged squared residuals, has a coefficient of -0.13495 with a probability value of 0.0001, indicating a significant positive association. This means that past shocks to the exchange rate have a substantial impact on current volatility, reflecting the persistence of volatility over time. In essence, the exchange rate experiences continued fluctuations as a result of previous disturbances in the market, highlighting the long-lasting effects of these shocks. The persistence in volatility is -0.13495 + 0.18562 + 0.89256/2 = 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 < 0.47162 <1. This is an indication of the high level of volatility persistence in the exchange rate of NGN/RUB. There is presence of leverage effect based on the sign, magnitude, and statistical significance of γ . This follows from the fact that the sign of 0.18562 is positive, that is, > 0; the magnitude |0.18562| is positive and also it has a p-value of zero. The conditions for non-negativity of variance equation coefficients, c > 0; $\alpha > 0$ 0, $\beta >= 0$ are satisfactory. The validity of the variance model was measured by 0.05067 which is ≥ 0 . Accordingly, the variance model is valid. Together, these results highlight the dynamic interplay between past market conditions and the current volatility of the NGN/RUB exchange rate. Also, the model fit statistics demonstrate a concrete capacity to explain the NGN/RUB exchange rate movements as made evident by the adjusted R² of 61% having accounted for the number of predictors.

Table 18 provides the estimates of the conditional volatility of the NGN/INR exchange rate. The ARCH(-1)² term, representing lagged squared residuals, has a coefficient of 0.704221 with a probability value of 0.0111. This indicates a significant and positive relationship, meaning past shocks to the exchange rate contribute

strongly to its current volatility. The persistence of volatility, as captured by this term, highlights the importance of recent market disturbances in shaping exchange rate fluctuations. Additionally, the GARCH(-1) term, which accounts for the lagged conditional variance, has a coefficient of 0.010364 and a probability value of 0.0001, indicating a highly significant effect. The degree of volatility persistence was estimated at 0.34811. This value is < 1 with the implication of the incidence of volatility persistence in the exchange rate of NGN/INR. Additionally, the statistical significance of 0.14246 indicates the impact of leverage. The conditions for nonnegativity of variance results are confirmed despite the negative α since 0.72355 > 0; $0.66307 \ge 0$, and $-0.10932 + 0.14246 = 0.03314 \ge 0$. Collectively, these results highlight the dynamic interplay between past shocks and volatility persistence in determining the NGN/INR exchange rate's conditional variance, with significant implications for risk management and forecasting in the foreign exchange market for both Nigeria and India. Also, the model fit statistics indicate a solid explanatory power for the NGN/INR exchange rate. An Adjusted R-squared value of 0.509970 suggests that approximately 51% of the variation in the NGN/INR exchange rate is captured by the variation in the independent variables. This is a confirmation of the model's reliability and robustness in capturing key drivers of NGN/INR exchange rate.

The estimates for the NGN/CNY exchange rate's conditional volatility as reported in Table 19 indicated a considerable persistence in volatility of 0.531137 < 1. There is a significant leverage effect as denoted by 0.15294. All conditions for non-negativity of variance equation coefficients are met, that is, 0.23597 > 0; 0.14623 > 0, 0.76315 ≥ 0 . The coefficient for the ARCH(-1)^2 term is 0.14623, with a highly significant probability value of 0.0000, indicating that past shocks to the exchange rate (residuals) have a strong and persistent impact on current volatility. This suggests that volatility exhibits autocorrelation, meaning that past volatility influences future volatility. The GARCH(-1) coefficient is -0.76315, with a probability value of 0.0006, indicating that past volatility positively and significantly affects current volatility. These findings jointly highlight the persistent volatility in the CNY/NGN exchange rate, which is crucial for understanding the dynamics of exchange rate risk and volatility forecasting. Also, having adjusted for number of degrees of freedom, the adjusted Rsquared value of 51% suggests that the model offers a moderate fit to the data, explaining a reasonable proportion of the NGN/CNY exchange rate's variation. This indicates that while the model captures some key factors driving the CNY/NGN exchange rate, there may be other unaccounted influences.

The volatility of the ZAR/NGN exchange rate as reported in Table 20. The constant of the variance equation is 0.177017, with a z-statistic of 2.878664 and a p-value of 0.0040, indicates a significant baseline level of volatility. The ARCH(-1)^2 term, with a coefficient of 0.14519 and a p-value of 0.0141, demonstrates significant short-term volatility crowding, where past shocks strongly influence current volatility. The GARCH(-1) component, with a coefficient of -0.69573, suggests that past volatility has a dampening effect on future volatility, confirming a mean-reverting pattern in ZAR/NGN exchange rate volatility. The statistical significance of these components underscores the model's robustness in capturing the dynamic nature of volatility in the exchange rate.

Overall, the ARCH (-1)² and GARCH(-1) terms in the variance equations across all currencies demonstrate statistically significant influences of past shocks and meanreverting tendencies, enhancing the reliability of VaR-GARCH methodology. According to the current results, exchange rate of the Naira in relation to each of the BRICS currencies (NGN/BRL, NGN/RUB, NGN/INR, NGN/CNY, NGN/ZAR) reacted positively and significantly to changes in one-period lagged value. Consequently, the GARCH model reveals critical insights into exchange rate volatility. The ARCH term as given by the coefficient of RESID(-1)² given as α is negative and significant at 5% level except for China and South Africa. The leverage effect as measured by the coefficient of RESID(-1)²*(RESID(-1)<0) given as γ was positive and significant all through the model indicating presence of leverage effect. With the exception of the NGN/ZAR exchange rate, the GARCH(-1) term was positive care able to deduce that the volatility is very persistent and aggregating because the GARCH coefficients are larger than the ARCH coefficient. When times of high volatility persist, the significant coefficients for lagged squared residuals effectively climaxes volatility crowding. The strength of the feedback effect of the Naira's volatility relative to the BRICS currencies is indicated by the volatility perseverance. By implication, even if to a lesser degree, future volatility shocks to the Naira exchange rate relative to the Real, Ruble, Rupee, Yuan, and Rand will be felt.

As a result, risk management may be impacted by the volatility in the dynamics of the exchange rate between the Naira and the BRICS currencies. Therefore, the persistence of this volatility in the dynamics of the exchange rate between the Naira and the BRICS currencies can affect risk management by causing financial market instability, mispricing assets, influencing how Nigerian businesses allocate their portfolios, and having a detrimental effect on economic activity and productivity. These findings agree with those of Chen et al. (2019) who deployed the GARCH models are useful tools for managing currency risk because they can adjust to deviations during times of crisis, even though they are compatible with the EMH in calm market conditions.

Based on the Efficient Market Hypothesis (EMH), the authors assessed how well the GARCH family models estimated the VaR of the EUR/USD exchange rate under various volatility regimes. Although there may be variations during times of market volatility, the EMH contends that asset prices accurately reflect all available information. The study concentrated on how well the EGARCH model captured two important aspects of exchange rate movements: asymmetric volatility and clustering effects. The findings of the present study showed a high prevalence of the leverage effect; for all models, the leverage parameter is positive and significant even at the 1% level. This suggests that when it comes to the Naira exchange rate in relation to the BRICS currencies, bad news has a greater effect than good news. Furthermore, negative shocks to the NGN/BRL, NGN/RUB, and NGN/INR have a greater effect on the conditional volatility projections than positive shocks, according to the negative ARCH(-1) coefficients of lag one. Nigeria's vulnerability to macroeconomic and financial market shocks, such as trade, monetary policy, output, external debt, and oil price shocks, is reflected in these dynamics.

Since the market value of the Naira/Real rate has large reactions to negative changes in oil prices, foreign debt, and positive changes in money supply; the study affirmed findings of Obuareghe et al. (2025) that changes in crude oil prices together with past values of broad money supply significantly and positively impact the exchange rate of the Naira in relation to foreign currencies. The results show that the Naira exchange rate against the Russian Rubble has significant and negative responsiveness to changes in output growth rate, BRENT crude oil prices and external debt levels. This aligns with the findings of Gainetdinova et al. (2024) where it was

established that the Rubble has significant vulnerability to oil prices. The findings also agree with the findings of Oyadeyi (2024) who reported monetary policy rate of the CBN, oil prices and financial development are all significant determinants of exchange rate volatility in Nigeria. The authors derived their results and findings from the ARCH, nonlinear GARCH and the ARDL model estimations.

The pricing of the Naira/Yuan has major sensitivities to negative fluctuations in BRENT crude oil prices, external debt, CBN policy rate and a positive output growth. The results of our research lend credence to the results obtained by Suhendra et al. (2022), Williams & Prasad (2019), Khan et al. (2019). Suhendra et al. (2022) reported that the discount rate of the central bank of Indonesia caused significant adverse variations in Indonesian exchange rate. Williams & Prasad (2019) discovered that net trade significantly caused the exchange rates of Japan and India to fluctuate, but only had marginal impact on the Yuan. The results of Khan et al. (2019) indicate that output growth and trade openness impacted positively and significantly the USD/CNY exchange rate while interest and inflation rates negative and significantly impacted the exchange rate. The valuation of the Naira/Rupee exchange rate is significantly responsive to the vulnerability of trade balance (deficit or surplus), external reserves, foreign debt, monetary policy rate, and BRENT crude oil prices. Hasan & Islam (2023) found that foreign exchange reserve; amongst other variables are the most influential determinants of exchange rate movements in Bangladesh. The results of the present research align with the with the findings of Ghauri et al. (2024) who based their findings on ARDL model estimation to conclude that interest rates, is amongst the significant determinants of the fluctuations in the exchange rate, noting that exchange rate positively reacted to interest rate changes.

The value of the Naira/Rand is highly susceptible to shifts in external reserves, debt levels, and the money supply, which are indicators of financial soundness. The results of our research lend credence to the results obtained by Ohaegbulem & Iheaka (2024) that the NGN exchange rate fluctuations respond significantly and positively to changes in external reserve, and the level of external debt. The findings align with those of Eduardo et al. (2024) who found that countries with enormous government debt suffered undervalued currencies. In terms of comparative analysis, in order to predict and quantify exchange rate volatility and

VaR, this study mainly estimated the VaR with the goal of increasing the precision of risk forecasting, and volatility modeling. With a macro-financial viewpoint, the study findings compare with those of Efuntade & Efuntade (2023) who analyzed long-term effects of oil price volatility on Nigeria's foreign exchange rate as well as those reported by Hashmi et al. (2022), and Ateba et al. (2024). These authors emphasize lopsidedness and country-specific dynamics in the analysis of heterogeneity of oil price shock impacts on currency rates in oil-exporting nations using the ARDL Bound test/error correction technique, using quantile ARDL and quantile-on-quantile regression (QQR) technique. Hashmi et al. (2022) highlights how oil prices and currency rates impact stock prices differently in bullish and bearish markets. The evidence of asymmetric oil shock impacts provided by Ateba et al. supports the need for nation-specific policy formulation in reaction to changes in the world oil price. Our results are similar to those obtained by Bouslama (2023) for dynamic BRICS stock-oil dependence based on VaR analysis.

5.1. Policy implications and recommendations

The necessity of developing country-specific policies in response to shifts in the global oil price is supported by the evidence of possible losses prediction during unfavorable currency fluctuations. Similarly, by contextualizing VaR within crisis moments and Nigeria/BRICS's currency rate-money supply and oil price variation connections, the study offers more direct insights into dynamic dependencies under financial stress. The oil-selling BRICS' predictive capabilities are ideal for risk management and market monitoring, while the macro-structural analysis and policy implications of the oil-dependent emerging nations offer the depth and context required for economic planning.

By implementing dynamic and data-driven VaR-based models for tracking the Naira's exposure to BRICS currencies, the Central Bank of Nigeria (CBN) and pertinent financial authorities can fortify their foreign exchange risk management frameworks, thereby reducing exchange rate risk and improving financial stability. In light of the substantial volatility concerns found in the research, particularly in bilateral exchange rate interactions with currencies like the Russian ruble and the Chinese Yuan, the following steps are advised:

- 1. Predict possible losses during unfavorable currency fluctuations which reflect a longer-term view of structural linkages, particularly among Nigeria's main trading and investment partners among the BRICS, use VaR-based stress testing to national foreign exchange reserves and financial institutions.
- 2. The Naira exchange rate vis-à-vis currencies of the BRICS share a vulnerability to oil shock and debt levels, emphasizing the need for robust economic management and dynamic forecasting models. The implications of the findings underscore the need for strategic interventions to enhance macroeconomic stability, mitigate external vulnerabilities, and foster sustainable exchange rate management in Nigeria.
- 3. Additionally, strengthening the regulatory framework to manage speculative activities is a key to reducing excessive volatility in the foreign exchange market. Traded currency markets, such as those involving the NGN/JPY and NGN/BRL pairs, are highly susceptible to volatility. Enhancing market depth and providing liquidity support through central bank interventions can stabilize these markets.
- 4. Addressing macroeconomic vulnerabilities, fostering economic diversification, and enhancing forecasting capabilities are essential steps to achieve exchange rate stability and economic growth in Nigeria. By adopting these policy measures, Nigeria can better navigate the complexities of global financial markets while safeguarding the value of the Naira.
- 5. Strengthening fiscal policies, enhancing trade balances, and mitigating external vulnerabilities can foster exchange rate stability and support informed decision-making for policymakers and stakeholders. To assist Nigerian importers, exporters, and investors in mitigating currency risk, promote the creation and accessibility of financial products such currency futures, forwards, and swaps.
- 6. Establish early causation system that initiate policy actions when currency risk thresholds are crossed, and build institutional capacity in financial institutions for real-time VaR monitoring. By providing significant causal findings and policy effects, this study highlights the necessity of enhanced foreign exchange management in developing nation like Nigeria.

6. Conclusion

This study evaluates exchange rate dynamics and VaR for BRICS currencies (BRL/NGN, RUB/NGN, INR/NGN, CNY/NGN, and ZAR/NGN) against the Nigerian Naira. The study reveals a mix of stability and volatility behavior for Naira against BRICS currencies influenced by domestic policies and external conditions. Accordingly, the CBN must consciously rebalance its exposure to BRICS currencies by employing risk-weighted analysis rather than merely trading volumes in view of the asymmetric risk. Also, the CBN should, for instance, seek more local currency settlement arrangements with the BRICS nations. This could lessen reliance on the US dollar and exposure to its volatility. These were made evident in the empirical influences exerted by macroeconomic factors such as financial healthiness measured as changes in broad money supply as a percentage of output (m2/gdp), trade balancedeficit or surplus, external reserves, monetary policy rate, changes in BRENT crude oil prices, and output growth rate in determining exchange rate movements. The MCS confirms the robustness of the risk metrics executed under the VaR analysis for the Naira exchange rate in relation to BRICS currencies. The analysis of the exchange rates of the Naira in relation to currencies of the BRICS reveals significant trends, volatility patterns, and predictive accuracy challenges. Periods of heightened global or domestic instability amplify uncertainty and forecasting challenges, as seen in post-2020 trends across all currencies. This study has made significant contributions to the understanding of foreign exchange forecasting and VaR estimation for the Nigerian Naira, particularly in relation to BRICS currencies. The research findings confirm that macroeconomic factors influencing exchange rate movements, including interest rate differentials, market volatility, and inflation rate differentials play a critical role in determining the exchange rate of the Naira against both major currencies and BRICS currencies, with varying degrees of impact based on currency pairs and market conditions. Interest rate differentials were found to have a positive and significant influence on exchange rates for some pairs. The generalizability of the findings may be restricted to the currencies of BRICS covered by the study. The results may not directly apply to other currencies due to differences in economic structures, policy environments, and global market dynamics. VaR estimations are effective. The

ANALYSIS OF VALUE-AT-RISK (VAR) OF NAIRA AGAINST BRICS CURRENCIES

estimations derived from the V-C, MCS, and H-S approaches, however, depend on the data as well as the pattern of distribution applied during the V-C and MCS processes. Since non-normalities are frequently seen in financial data, including exchange rates that reveal a variety of mean, standard deviation, skewness, and tail features; V-C, MCS, and H-S approaches may produce entirely different VaR estimations. Future research could explore the application of machine learning algorithms, such as neural networks and deep learning models, to forecast exchange rates and assess risks, particularly in highly volatile markets. These methods could capture non-linear interactions and adapt to new data patterns, offering a more dynamic and adaptable approach to forecasting. Having performed the MCS and H-S as a check of robustness on the V-C technique in the computation of VaR, this study contributes valuable insights into exchange rate forecasting and exchange risk valuation, providing a foundation for further research and practical applications in emerging markets. Thus, further research is necessary to refine and expand upon the findings of this study. This provides deeper intuition into the complexities of foreign exchange forecasting and risk management in emerging markets like Nigeria, contributing to more robust economic policy formulation and improved financial decision-making.

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Variables	Critical Values 5%	ADF T-Statistic	Order Of Stationary	Remark
EXCR	-2.909206	-8.165319	I(1)	Stationary
m2/gdp	-2.909206	-7.759645	I(1)	Stationary
Tbal	-2.909206	-6.585367	I(1)	Stationary
ERev	-2.909206	-7.836475	I(1)	Stationary
Opr	-2.909206	-13.25672	I(1)	Stationary
OgR	-2.909206	-9.289482	I(1)	Stationary
Mpor	-2.909206	-8.051474	I(1)	Stationary

Appendix 1. Unit root results for Naira/Real exchange rate

Source: authors' estimation results (2025).

Appendix 2. Unit Root Results for Naira/Rubbles exchange rate

Variables	Critical Values 5%	ADF T- Statistic	Order Of Stationary	Remark
EXCR	-2.909206	-8.194029	I(1)	Stationary
m2/gdp	-2.909206	-7.818122	I(1)	Stationary
Tbal	-2.909206	-6.585367	I(1)	Stationary
ERev	-2.909206	-7.851219	I(1)	Stationary
Opr	-2.909206	-6.234728	I(1)	Stationary
OgR	-2.909206	-8.129365	I(1)	Stationary
Mpor	-2.909206	-10.546879	I(1)	Stationary

Source: authors' estimation results (2025).

Appendix 3. Unit root results for Naira/Rupee exchange rate

Variables	Critical Values 5%	Adf T-Statistic	Order Of Stationary	Remark
EXCR	-2.909206	-3.572430	I(1)	Stationary
m2/gdp	-2.909206	-5.902116	I(1)	Stationary
Tbal	-2.909206	-6.585367	I(1)	Stationary
ERev	-2.909206	-7.916867	I(1)	Stationary
Opr	-2.909206	-10.36192	I(1)	Stationary
OgR	-2.909206	-9.267913	I(1)	Stationary
Mpor	-2.909206	-8.156739	I(1)	Stationary

Variables	Critical Values 5%	ADF T-Statistic	Order Of Stationary	Remark
EXCR	-2.909206	-7.748633	I(1)	Stationary
m2/gdp	-2.909206	-7.759613	I(1)	Stationary
Tbal	-2.909206	-6.585367	I(1)	Stationary
ERev	-2.909206	-7.906332	I(1)	Stationary
Opr	-2.909206	-10.32874	I(1)	Stationary
OgR	-2.909206	-8.193675	I(1)	Stationary
Mpor	-2.909206	-9.872196	I(1)	Stationary

Appendix 4. Unit root results for Naira/Yuan exchange rate

Source: authors' estimation results (2025).

Appendix 5. Unit root results for Naira/Rand exchange rate

Variables	Critical Value 5%	ADF T- Statistic	Order Of Stationary	Remark
EXCR	-2.909206	-8.211739	I(1)	Stationary
m2/gdp	-2.909206	-7.747689	I(1)	Stationary
Tbal	-2.909206	-6.585367	I(1)	Stationary
ERev	-2.909206	-7.970241	I(1)	Stationary
Opr	-2.909206	-6.309256	I(1)	Stationary
OgR	-2.909206	-9.672058	I(1)	Stationary
Mpor	-2.909206	-7.292540	I(1)	Stationary

Source: authors' estimation results (2025).

Appendix 6. Co-integration test results for NGN/BRL exchange rate

Co-integrating			Critical value	
Equations	Eigenvalue	Trace Statistic	(5%)	P-Value
None	0.280192	68.87248	47.85613	0.0014
At most 1	0.176491	18.81742	29.79707	0.5061
At most 2	0.090892	6.972401	15.49471	0.5809
At most 3	0.018831	1.159623	3.841466	0.2815

Co-integrating			Critical value	
Equations	Eigenvalue	Trace Statistic	(5%)	P-Value
None	0.334146	49.41380	47.85613	0.0016
At most 1	0.154628	19.60604	29.79707	0.4500
At most 2	0.130483	9.359375	15.49471	0.3332
At most 3	0.013523	0.830502	3.841466	0.3621

Appendix 7. Co-integration test results for NGN/RUB exchange rate

Source: authors' estimation results (2025).

Appendix 8. Co-integration test results for NGN/INR exchange rate

Co-integrating			Critical value	
Equations	Eigenvalue	Trace Statistic	(5%)	P-Value
None	0.315846	67.56857	47.85613	0.0007
At most 1	0.227067	22.41469	29.79707	0.2760
At most 2	0.100212	6.703371	15.49471	0.6124
At most 3	0.004286	0.261996	3.841466	0.6087

Source: authors' estimation results (2025).

Appendix 9. Co-integration test results for NGN/CNY exchange rate

Co-integrating			Critical value	
Equations	Eigenvalue	Trace Statistic	(5%)	P-Value
None	0.287983	50.87775	47.85613	0.0026
At most 1	0.215995	23.15889	29.79707	0.2383
At most 2	0.122248	8.315121	15.49471	0.4323
At most 3	0.005905	0.361263	3.841466	0.5478

Source: authors' estimation results (2025).

Appendix 10. Co-integration rest results for NGN/ZAR exchange rate

Co-integrating			Critical value	
Equations	Eigenvalue	Trace Statistic	(5%)	P-Value
None	0.314706	50.00466	47.85613	0.0010
At most 1	0.284865	26.95234	29.79707	0.1028
At most 2	0.092178	6.500016	15.49471	0.6364
At most 3	0.009803	0.600915	3.841466	0.4382