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Dependence of decisions made under risk on the skewness of outcome distribution: experimental results

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Abstract

Aim: The aim of the article is to evaluate the influence of skewness of outcome distribution on the willingness to take risk in a specific lottery.
Methodology: In the article we describe the results of an experiment conducted among students of a higher education institution. During the experiment we verified the influence of skewness of outcome distribution on the decisions to participate in a lottery whose outcome affected positively or negatively the amount of points which the student could gather in order to be awarded with a credit for a class.
Results: The experiment showed that while the outcome distribution is negatively skewed, people are less risk-loving, and when it is positively skewed people tend to take more risk (i.e. take part in a lottery).It is important to underline that each of the proposed lotteries was characterized by the same expected value. Moreover, the result of the game might have been positive or negative.

Keywords: experiment, skewness of outcome distribution, risk
JEL: D110; D810

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Introduction

We could venture to say that every decision people make is done so in the presence of risk or uncertainty. The outcomes of those decisions may have more or less significant consequences from the economic, health and social point of view. Determinants of daily decision making may be found in personal characteristics (a varied level of knowledge, varied tendency to take risks, etc.), as well as in external conditions referring to the effects of potential decisions, access to information, etc.
Nevertheless, in every case when it comes to making a decision, there is a decision process unfolding which is the foundation for scholarly research. Research on the decision making process is ongoing, which indicates that the process has not been fully recognized. The diversity of the

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decision-making determinants and their influence on the decisions made, for example, on the tendency to take risk, causes problems in terms of constructing a study which would be adequate for isolating a single factor affecting decision- making. This is precisely the reason why researchers carry out experimental studies and/or surveys which are to identify the factors influencing the decision making in the presence of uncertainty or risk.

The aim of this paper is to evaluate the influence of skewness of outcome distributions on the tendency to take risk by players. On the basis of the experimental study with the participation of students, the hypothesis was verified that, "positive skewness of gains is accompanied by a greater tendency to take risks, whereas the negative skewness of gains is associated by a greater risk aversion". The confirmation of this hypothesis would explain why people are more willing to play e.g. a lottery when there is the so called "jackpot" (apart from the influence of the factor, being the value of the expected winning).

Literature Review

Bernoulli (1738)[1] found that the expected utility of the prize won in a game is more important than the value of the expected prize from this game. According to Bernoulli, the same game can have different values of utility for different players, which was proven empirically by, for example, Dohmen (2005), who, on the basis of a survey carried out on 22000 Germans, demonstrated that there was a significant link between taking risk and the person's age, gender and education level. Since it is very unlikely that the utility function of wealth is linear, being rather logarithmic, people are not willing to take risk (although Rabin and Thaler (2001) assert that in some cases risk aversion may also be explained by a concave utility function of wealth). Many years later Bernoulli's hypothesis was named Expected Utility Theory – EUT and was axiomatized by Neumann and Morgenstern (1944). Since then, according to the theory of decision making in the presence of uncertainty, it has been assumed that people aim at maximizing the expected utility, yet not the expected value. However, numerous scientists do not accept the EUT to be the correct theory for explaining the decision-making in the presence of risk (see, e.g., Markowitz, 1952).

Beside objective factors, such as, for example, the probability that a given situation or its outcome will occur, the decisions made in the presence of risk are also influenced by other factors.

Friedman and Savage (1948) argue that the curvature of the utility function depends on the amount of accumulated wealth, which affects the propensity to take risk. Thus, to give an example, people with low income (and little accumulated wealth) enjoy gambling when there is a low probability of winning a high prize. For people with average earnings (and average wealth) all kinds of gambling are attractive, whereas for people with high income (much accumulated wealth) gambling which offers a high probability and a small prize tends to be attractive. Studies conducted by P.J.H. Schoemaker and H.C. Kunreuther (1979) also refer to the influence the level of wealth exerts. Within a group of students they studied, those individuals who behaved in a more risky manner were at the same time characterized by lower earnings and less wealth than the individuals who preferred safe choices. Those findings, however,

[1] His work *Specumen theoriae novae de monsura sortis* was translated in 1954 and published in the journal Econometrica under the title *Exposition of a New Theory on the Measurement of Risk.*

found no confirmation in any other group they studied, that is, amongst clients of some insurance company. Hoffmann, Henry, Nikos and Kalogeras show that decisions made by people depend on two factors. One of them is the level of wealth (at various life stages), that is what has already been accumulated, with the other factor being individual aspirations, which we will describe further on in the paper (A. O. I. Hoffmann, S. F. Henry, N. Kalogeras, 2013).Moreover, Kahneman and Tversky (1979) argue that it is not the level of wealth that influences the attitude towards risky situations, but rather the changes in one's wealth seen against a certain point of reference (this is the assumption of the so called Prospect Theory).

As already mentioned, it is possible that aspirations, that is, the minimum returns that an individual must/wants to achieve, can influence the decision-making in a given risky situation. Among others, this is confirmed by Sokołowska's research (2006). She demonstrates that many people while changing their level of aspirations, change their preferences with respect to the lottery they would like to take part in, whereas the change of the aspiration level does not affect risk estimation. Moreover, the studies by Hoffmann, Henry and Kalogeras (2013) suggest that, with respects to games where multiple decisions are made, the level of aspiration of individual persons provides their main point of reference at early stages of the decision-making process, while their initial possession (the level of wealth at the beginning of the experiment) becomes the focal point of reference at later stages of the decision –making process.

The so called "peanuts effect" is yet another example of a factor influencing decision-making. It says that people have a greater tendency to take risk when they deal with smaller bets, while their risk aversion increases when the bet is higher (although clear results could only be obtained for the decisions made on gains, see: Mitchell and Wilson, 2010; Hogarth and Einhorn, 1990).

On the other hand, it appears that the higher the possible gain, the more attractive the lottery (Forrest et al., 2002). Yet, in this situation the expected value changes, which could be the factor determining whether or not the lottery is attractive. Some scholars therefore maintain that this relationship may be linked to the preference for games which are characterized by a positive skewness of the outcome. Åstebro et al. (2014), and Garret and Sobel (1999), on the basis of their study, conclude that individuals who decide to participate in lottery games which are subject to much risk are actually not so much prone to risk, but it is rather that their decisions are motivated by the preference for a positive skewness of gains distributions in the game (they are called "skewness-lovers"). Patton (2004) points out that the greater the negative skewness of the outcome distribution, the smaller tendency to take risk amongst those making decisions under risk. M. Palenik (2014) failed to confirm his hypothesis, while conducting his study, which states that "games with positive skewness are more attractive than the ones with negative skewness". Yet, it should be stressed here that the games whose attractiveness was evaluated differed from one another in terms of the expected value, which could have affected the results.

The conclusion made on the basis of the presented studies concerned with the influence of the skewness of the outcome distributions may explain people's tendency to gamble, even if the expected value is negative. Further to that, the value of the top prize (as, for example, in Lotto lottery game with a asymmetrical outcome distribution)

may have an influence on the decision whether or not to play to a greater degree than the expected value of gains (as is the case for rollover jackpots). In the study, the expected values of outcomes in the games were the same and the games differed in the skewness of the outcome distribution (and consequently, in that the prize varied)

The Study

The study was carried out amongst 376 students in the II and III year of the undergraduate studies at Wrocław University of Economics and WSB University in Wrocław.
The participants were divided into three groups: 112 persons (GROUP I), 129 persons (GROUP II) and 135 persons (GROUP III). In each group the study was carried out differently, which will be outlined below.

Questionnaire
The questionnaire comprised 12 lottery offers in which one could gain or lose, with a given probability, a specified number of points which were added to or subtracted from the points earned during the entire semester. The expected value of the prize for each lottery was the same and equaled 2.5 points. However, the lotteries differed in terms of their diversity and the strength and direction of skewness of the number of points won. Each lottery proposal contained information on how many points one could lose or win and on the probability of getting the results which were defined in the lottery, depending on the roll of a dice (it was shown, how many dots one must have with one roll in order to realize a particular outcome). The probability was not specified in numbers because previous studies showed that the surveyed were little susceptible to the probability expressed in numbers (Forlicz et al., 2014).
What the questionnaire given to the surveyed group looked like shows fig. 1.
The individual lotteries included in the questionnaire should be interpreted as follows: in lottery 1, to give an example, a person could lose one point if she got 1,2 or 3 pips at the roll of a dice; the number of the points she accumulated would not change if she got 4 or 6 pips, while a throw getting 6 pips would allow her to gain additional 18 points.
Filling in the questionnaire involved marking with an x the „YES" box on the left side if the surveyed individual was willing to participate in this kind of lottery or to mark the box with "NO" on the right side if he/she did not want to participate. Lottery 7, in which one could lose no points, was included in the questionnaire so as to check if the person giving answers understood sufficiently well the decision-making problems he/she was facing. The questionnaires of persons who did not want to play lottery 7 were deemed invalid and their results were not included in the analysis.
The survey in individual groups unfolded as follows. In all the groups under discussion the same rules of receiving credits applied. During the semester one could earn maximum 40 points and in order to get a credit one should accumulate 20 points. The study was conducted at the very beginning of semester when the students had not had any points accumulated. During two weeks, the meetings with the surveyed individuals were arranged in smaller subgroups; however, never was there any mixing of participants from the different groups. The study was carried out according to two scenarios:

- Scenario A - hypothetical

GROUPS I and II were informed that the researchers would like to know the participants' preferences with respect to decision-making under risk and that

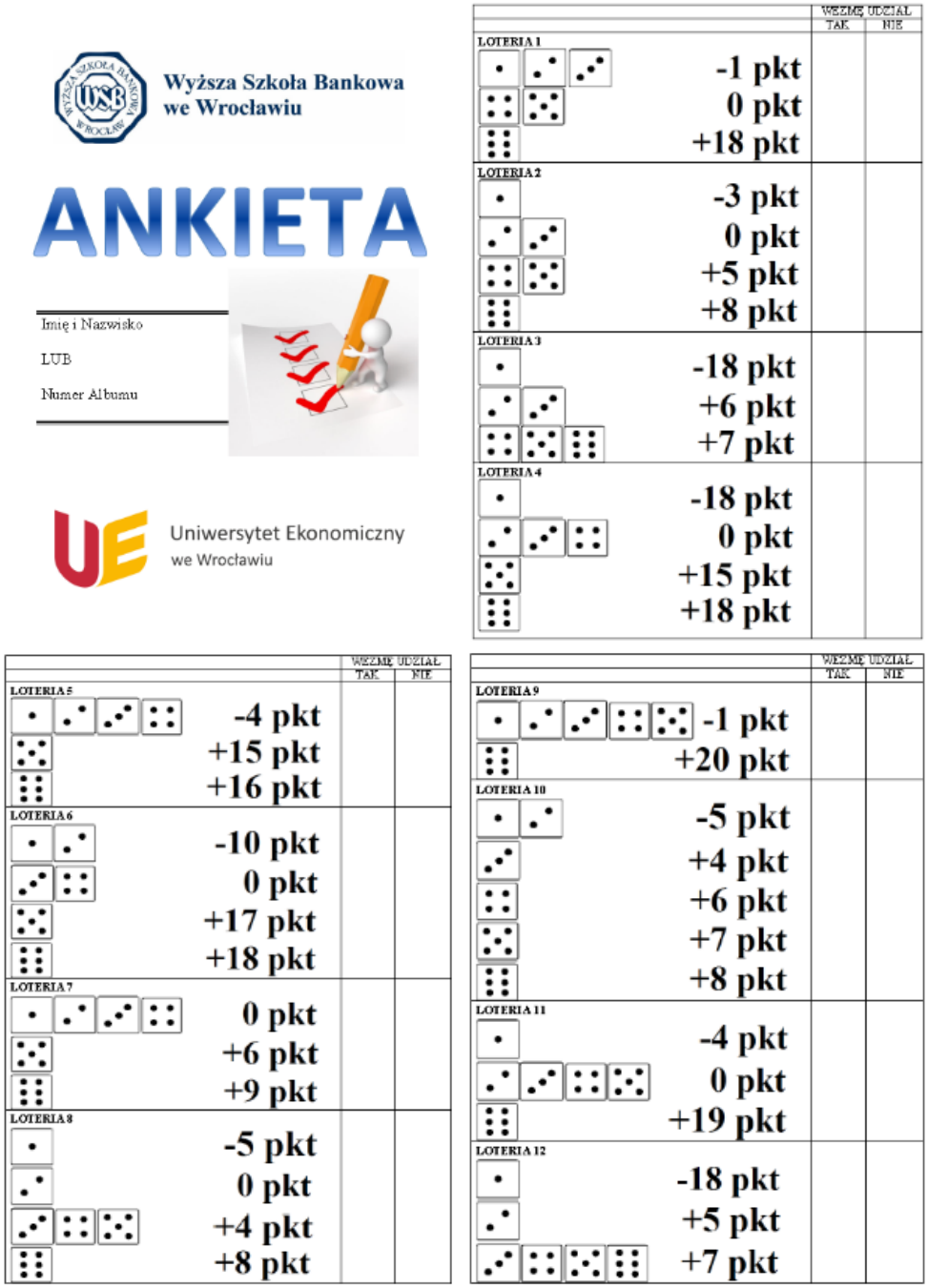

Fig 1. The questionnaire including the description of individual lotteries.

Source: Authors' own study.

was why the survey was conducted. It was emphasized that the questions contained in the questionnaire were purely hypothetical, however, respondents were asked to provide answers according to their beliefs for that was important for science. Moreover, the surveyed were promised that by taking part in the study they would have the opportunity to gain later additional points for credits. The promise was fulfilled in that the respondents from GROUP I were offered the opportunity to play for real points by participating in lottery 5, and those from GROUP II were given the opportunity to participate in lottery 8. These lotteries were selected as lotteries which are characterized by a similar strength of skewness, yet with a different direction. Persons who were willing to participate in the real lottery were asked to enroll themselves in the list and not before all of them made up their mind and enrolled in the list, were they asked to throw a dice.

- Scenario B – the real one

Persons appointed to GROUP III also heard that it was about a scientific study, with the difference that their decisions were not to be purely hypothetical. The surveyed were informed that one of the decisions they were going to take while filling in the questionnaire (the same questionnaire as for GROUPS I and II) would become a real decision (which lottery that was supposed to be was decided by an additional drawing).

In order not to influence the results of the survey by providing different explanation as to the rules of the experiment, at the beginning the instructions were read to the participants appointed to GROUPS I and II.

Study' Results

Table 1 presents the results of the experiments conducted. The table shows the percentage of individuals who stated their willingness to participate in a particular lottery, and the individual categories correspond to the game scenarios. The last column demonstrates the coefficients of the skewness of the outcome distributions for the individual lotteries. The skewness coefficients are calculated according to the following equation:

$$v = \frac{\sum_{i=1}^{n} (x_i - m)^3 \cdot p_i}{s^3}, \quad (1)$$

where:

x_i – i-th value of the prize in the lottery

m –, expected value of the prize in a given lottery

p_i – probability of achieving the i-th value of the prize in the lottery

s – standard deviation of the lottery prize.

The intra-group analysis was carried out by comparing the decisions made by persons ascribed to GROUP I or II and by those to GROUP III. Table 1 compares the percentage of persons who decided to play under hypothetical circumstances and in the situation when one of the decisions was to be binding (that is to bring a real effect). Also, the information on what hypothetical decisions were made by persons from GROUP I and II who stuck to their decisions in the real situation was added to the table.

Chi-squared test revealed that the answer distributions vary when comparing the decisions made by GROUP I and II (combined) with the decisions made by GROUP III (p value 0.012237). However, there are no significant differences in the distributions of answers by persons from GROUP I and II who were consistent in their choices (scenario A; 5 or 8 confirmed by the real decision taken after filling in the questionnaire) and the answers by persons from GROUP III (scenario B). The rank correlation coefficients between the percentage

of persons deciding to play in both cases, for whom goodness of fit was tested, were high and equaled slightly over 0.95. Despite the fact that a portion of the surveyed answered "hypothetically", and some "really", their decisions should be considered as compatible.

Table 1 Percentage of persons who decided to participate in a particular lottery by game scenario

	PERCENTAGE OF PERSONS WHO DECIDED TO PLAY					Skewness coefficient
Lottery number	Scenario A, 5 real	Scenario A, 8 real	Scenario A (5 and 8 combined, real)	Scenario A (5 and 8 combined real, confirmed)	Scenario B	
3	18%	14%	16%	15%	9%	-1,78
12	18%	17%	18%	19%	14%	-1,76
10	56%	58%	57%	53%	37%	-1,01
8	46%	43%	44%	50%	32%	-0,64
4	30%	21%	25%	25%	16%	-0,30
2	72%	76%	74%	72%	56%	0
6	13%	17%	15%	15%	8%	0,32
5	24%	36%	30%	26%	10%	0,71
11	51%	46%	48%	52%	31%	1,63
1	78%	85%	82%	84%	85%	1,77
9	61%	73%	67%	71%	66%	1,79
Number of observations	104	121	225	139	131	

Source: Authors' own elaboration based on their own studies

Table 2. Correlation coefficients between the skewness coefficient of the lottery and the percentage of persons willing to participate in the lottery, by experimental groups.

GROUP	**Correlation coefficients between the skewness correlation of the lottery and the percentage of persons willing to play a particular lottery**	
	Pearson's linear correlation coefficient	**Spearman's rank correlation coefficient**
Scenario A (5 and 8 combined real)	0.5998	0.5363
Scenario A (5 and 8 combined real, confirmed)	0.6207	0.5614
Scenario B	0.6016	0.4545

Source: Authors' own elaboration based on their own study

With the view to verify the hypothesis advanced in the introduction, stating that positive skewness of the outcome is accompanied by a greater tendency to take risk, whereas negative skewness is accompanied by a greater aversion to risk (to eliminate other factors, the expected value was the same for all the lotteries and was equal to 2.5 points), Pearson's linear correlations coefficients and Spearman's rank correlation coefficients were calculated between the percentage of persons willing to play a particular lottery and the skewness coefficient (see table 2). Spearman's rank correlation coefficients range between 0.4545 and 0.5614. Higher coefficients were observed using Pearson's linear correlation coefficient because the nature of the relationships is closer to the linear one, and its value varies between 0.5998 and 0.6207. The p value for the significance test of the correlation coefficient for the above Pearson's correlation coefficients ranges from 0.041 to 0.051, which indicates that they can be considered to be statistically significant. Both Pearson's linear correlation coefficients and Spearman's rank correlation coefficients are positive, which shows that the higher the skewness coefficient value, on average, the greater the number of people willing to participate in a given lottery. We may thus conclude that positive skewness of outcome distributions has a positive effect on the decision to participate in the lottery made by surveyed persons, which implies that it is accompanied by a greater tendency to take risk. However, in order to be able to confirm this finding, more lotteries should be offered to the surveyed so as to calculate the correlation coefficients for a greater number of lotteries and then test again the correlation significance.

Conclusion

In this paper, the attempt was made to evaluate the influence of skewness of outcome distributions on risky decision-making. The factors affecting risk taking by individuals have been outlined extensively in literature, yet they have not been sufficiently investigated. One of the difficulties is to isolate the impact every factor has on decision-making. In similar studies, authors also tended to change the value of the expected prize by "manipulating" the skewness of outcome distributions, which may have influenced their final results. In the experiments conducted by the authors of the present paper, only the skewness of outcome distribution and the size of the prize were changed.

The studies confirm to a certain degree that a smaller risk proclivity occurs with negative skewness of outcome distributions in the game. Linear correlation coefficients between the percentage of individuals willing to play a particular lottery and skewness coefficient equaled roughly 0.6 (and they were significant statistically at the level of 0.05).

On the other hand, with one-time real choices, more respondents were willing to participate in the lottery with negative skewness than the positive one. However, having analysed this part of the study, the authors concluded that some individuals made decisions under the influence of the group, which may have distorted the results obtained. Another constraint, which has already been mentioned, is the relatively small number of the lotteries proposed due to temporal limits, as the experiments were carried out during seminars, which ensured the possibility to survey individuals in the number defined in advance.

The findings do not explain the relationships outlined here in their causal sense, nevertheless they show that, amongst students under study, there is a relationship between tendency to take risk and prize skewness, and that this relationship in its nature is close to the linear one.

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Asymetria w rozkładach wygranej a decyzje podejmowane w warunkach ryzyka w świetle badania eksperymentalnego

Abstrakt

Cel: Celem artykułu jest dokonanie oceny wpływu asymetrii rozkładu wygranej na skłonność do podejmowania ryzyka.

Metoda badawcza: W artykule zaprezentowano wyniki przeprowadzonego wśród studentów uczelni wyższej eksperymentu. W trakcie eksperymentu sprawdzano wpływ asymetrii rozkładu wygranej na decyzje o udziale w grze losowej, której wynik wpływał pozytywnie lub negatywnie na liczbę punktów, które student mógł zgromadzić w celu zaliczenia przedmiotu.

Wnioski: Przeprowadzone badanie wykazało, że mniejsza skłonność do ryzyka występuje w przypadku lewostronnej asymetrii wygranej, natomiast większa skłonność do ryzyka występuje w przypadku prawostronnej asymetrii wygranej. Zaznaczyć trzeba, że w każdej z proponowanych gier wartość oczekiwana wygranej była taka sama. Ponadto wynikiem gry mogła być zarówno wygrana, jak i przegrana.

Słowa kluczowe: eksperyment, asymetria rozkładu wygranej, ryzyko