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Extreme Statistics in the Analysis of the Exchange Rate Volatility of CHF/PLN

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Abstract

The article presents the analysis of the exchange volatility of the Swiss franc against the Polish złoty (CHF/PLN). This issue has a significant practical relevance. Owing to its considerable volatility, the exchange rate of this currency pair has had a significant impact on the situation of several hundred thousand of borrowers. In the first decade of this century there were numerous loans (predominantly housing loans) granted which were denominated in Swiss francs. This often causes serious difficulties in debt servicing and can have significant social and economic implications. Therefore, those in power have been seeking to solve this problem in a variety of ways although no solutions has been implemented at the time of sending this paper for publishing.

Introduction

From a legal point of view, borrowers raise the impossibility of predicting the volatility of the exchange rate of the CHF/PLN pair. This was often accompanied by bankers' assurances as to the benefits of the foreign exchange loan (interest rate, low risk). Further to that, there were cases where loan applications for loans in the Polish złoty were refused on the ground of the lack of creditworthiness. Lower loan installments in the Swiss franc could have positive effects for some borrowers in that they would obtain a positive credit-worthiness assessment. In such situations exchange rate risk was usually underestimated. The borrowers also raise the absence of real payments out (payments in) in Swiss francs. From an economic point of view, the occurrence of Swiss notes and coins is obviously of no importance. In this case, however, at least two kinds of lenders can be distinguished.

The first of them grants a loan in Swiss francs transferring the equivalent amount of the loan in the Polish

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złoty at the bank's buying rate. Thus, the bank, apart from granting the loan, sells the borrower złotys while buying francs. At the moment of the payment of the loan installment the borrower pays in Polish złotys. They pay the equivalent of the installment expressed in francs after converting them into złotys at the sell rate. For the first kind of borrowers, the spread was to be a hedge against the exchange rate volatility. Not before 2010 could borrowers (thanks to the court judgments) repay their debts in the Swiss franc, which made the risk for both sides symmetric.

The second kind of lenders enters into financial commitments (usually towards the parent bank) in Swiss francs at the moment of signing a loan agreement. The duration of those liabilities is similar to that of the credit claims. In this way, the bank eschews the risk relating to the exchange rate volatility. The entire exchange risk is borne by the client. The potential spread applied at the payment of installments provided the bank with an extra income.

In both cases the risk relating to the exchange rate volatility fell on the borrower. The essential question posed in this paper refers to the ability to predict the exchange rate of the CHF/PLN currency pair. While analyzing the exchange rate of this currency pair, the conclusion to be made is that the position of the Swiss National Bank was often decisive on this matter. Yet analysts have been unable to create a model based on this bank's (frequently) unpredictable decisions.

In this paper we will demonstrate a stochastic model based on the analysis of order statistics distribution. From the borrower's point of view, the maximum exchange rate in a given period is crucial for the ability to service debt. The analysis of the random variable distribution will therefore be significant: $\max\ X_1, X_2, ..., X_n$, where X_i

is the exchange rate of the CHF/PLN currency pair at moment i.

Two models will be demonstrated in the paper. The first one assumes stationary distribution and normal distribution of the currency pair under study. It is easy to criticize such an assumption, yet, it appears that even using such a simple model we can obtain interesting estimates of probabilities. At the time of the loan agreement conclusion the parties to the agreement often make exactly those kind of assumptions

Moreover, a model will be discussed where one moves away from the assumption of stationary distribution while maintaining the assumption of normal distribution. In both models one allows for the dependency of individual rates over time, which obviously reflects the real situation.

Extreme Statistics and the Models Applied

In the paper well known theorems on order statistics (particularly the extreme ones) will be used. The first findings in this field date back to the 1920s. While assuming the normality of the variables analysed and asymptotic independence, extremal types theorem will be useful here [Leadbetter, Rotsen, Lindgren, 1986] or [Galambos, 1978].

Since the normal distribution function belongs to the domain of attraction of the so called Type I (double exponential distribution), the theorem holds true (Leadbetter, Rotsen, Lindgren, p. 25).

Theorem 1.

If ξ_n is a sequence of independent normally-distributed random variables, then the random variable $M_n = \max\ (\xi_1, \xi_2, ..., \xi_n$ has a Type I distribution, that is:

$$P\{a_n(M_n - b_n) \leq x\} \rightarrow \exp(-e^{-x})$$

where:

$$a_n = (2 \ln n)^{1/2} ; \quad b_n = a_n - \frac{\ln \ln n + \ln 4\pi}{2 a_n}.$$

The above theorem has numerous variations and it seems that applying it directly to the analysis of currency exchange rates is impossible because of the fairly obvious dependency of individual exchange rates over time. In literature, however, there is evidence that constraining assumptions can be overridden (David, Nagaraja, 2003).

The first results of this type were obtained by Berman who argued that the condition $r_n \ln n \rightarrow 0$ is sufficient to maintain the thesis of theorem 1. Also, many other alternatives have been demonstrated for the description of dependencies. However, for currency exchange rates, those conditions cannot be verified because we have only one observation in a given moment. Estimating the correlation coefficient with the stationariness assumption leads, on the other hand, to (at the most) monotonicity. The existence and the border value cannot be verified. Therefore, the model of the distribution of maximum from the first theorem is a probable model, yet at the same time only a proposal which the parties to the loan agreement might be presuming.

While employing theorem 1, the probabilities of occurrences will be estimated at interesting moments in time. Thus, this is not a model aspiring to describe the phenomenon which is being analysed at every moment in time. The rule that applies here is rather the one deployed in warning forecasts. In this case it is about indicating the probabilities that certain unfavorable events will occur. Econometric models, the trend analysis or time series methods should therefore be rejected. They often use the sum of squares with or without weights as a criterion of the goodness of fit. In the latter case it means applying the same relevance to all the moments in time that are being analysed.

For estimating maximums, relevant values are interesting on certain intervals. The distribution of (suitably normalized) variable $\max_{t \leq n} X(t)$ is anlaysed, where $\max_{t \leq n} X(t)$ is the exchange rate of the currency pair under discussion. Observing the sequence of realizations we can conclude that in a given period one exchange rate (which occurred one time or several times) is maximum. This simple observation does not, however, refer to the distributions. If that was to be the case, then $\max_{t \leq n} (X(t))$ distribution would be identical with the distribution of one of the random variables $X(t)$. The thesis of the theorem I shows that in general this does not occur although an interesting fact to note is that the distribution function of the type I distribution belongs to the domain of attraction of the type I distribution function. In this case variables $X(t)$ have the same type of distribution as $\max_{t \leq n} X(t)$. A similar situation refers to the other two types of limiting distribution functions. The lack of differentiation between the maximums for the realizations (numbers) and maximums for random variables led to a number of misunderstandings outlined, e.g. in (Feller, 1978). For this reason, a simple example 1 is presented which illustrates the fact that $\max (X_1, X_2)$ has no distribution corresponding to any of the variables X_1, X_2

Example 1.
Consider an experiment which consists in throwing a dice (cuboidal) one time. Let X_1 denote a random variable – the score on the first die, X_2 - the score on the second die, but a random variable $Z = \max (X_1, X_2)$ has the following distribution:

Z	1	2	3	4	5	6
p	1/36	3/36	5/36	7/36	9/36	11/36

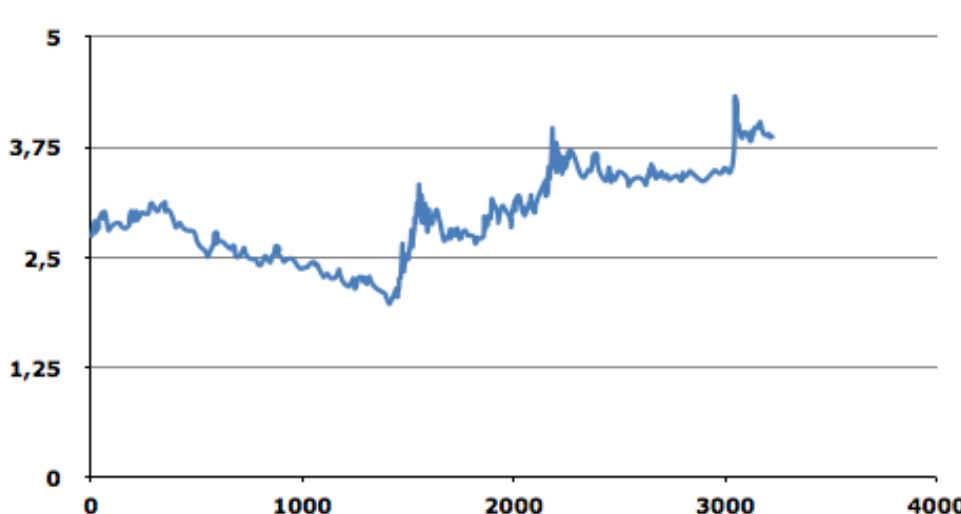As can be gleaned from the above, it is different from the uniform distribution in the set $\{1,2,3,4,5,6\}$.

Fig 1. The Swiss franc exchange rate from 02.01.2003 to 05.10.2015

Source: Authors' own study based on http://www.nbp.pl/home.aspx?f=/kursy/kursy_archiwum.html

Model 1

Many theorems of the order statistics (extreme) theory can be applied to the issue under discussion. The paper demonstrates the application of possibly simple tools. At the same time, other possible ways of analyzing this issue have been indicated. Three significant maximums can be noticed on the chart of the currency pair CHF/PLN. After having reached the significant maximums, the exchange rate stabilized slightly below the current maximum, but usually above the previous maximum. Therefore, reaching a high exchange rate usually implies a relatively permanent increase of the borrower's debt.

The observations started in 2003 (2 January). In order to obtain the conformity with the theorems applied, the numbering of days was employed instead of a direct application of dates. The days were numbered with natural numbers. On the day marked with

the number 1551 (18 January 2009) the exchange rate of the currency pair under study is equal to 3,3167. In this case the decision was to build a model based on a period which clearly precedes the increase trend resulting in reaching the limit of 3,3167. It was decided (considering the implied stationariness) that a good period in time will be the first 1408 days, starting from the very beginning of recording the exchange rates (2 January 2003) to 25 July 2008, when the exchange rate was 1,9741, thus historically being almost the lowest. This period was selected subjectively, but certainly not in a biased way. There were no signs that this trend might be reversed. During this period, constants were estimated necessary to standardize variable $X(t)$.

In the model, $X(t)$ denotes the NBP exchange rate of the CHF/PLN currency pair. It was estimated that the mean value equals $\hat{m}(1408)=2,5921$, while the standard deviation equals

$s(1408)=0{,}2847$. Those numbers were employed to standardize variable $X(t)$, obtaining variable $\eta(t)=\dfrac{X(T)\hat{m}}{s}$. The next step involved the calculation of normalizing constants a_n and b_n of the variable:

$$\max_{t\leq1551}(\eta(t))=M(1551)$$

according to the equations:

$$a_n=(2\ln n)^{1/2}$$

$$b_n=a_n-(2a_n)-1(\ln ln\ n+\ln4\pi)$$

For $n=1551$, $a_{1551}=3{,}8332$ and $b_{1551}=2{,}6528$ were obtained.
Next, the probability was estimated:

$$P(a_{1551}\ (M(1551)-b_{1551})\leq-0{,}4116)=0{,}221086$$

The constant -0,4116 is the realisation of the left side of the inequality. This means that the probability that the CHF/PLN exchange rate will exceed the level of 3,3167 by the 1551st day is equal to $1-0{,}221086\cong0{,}78$. In this case, forecasting the level of 3,3167 performed on 25 July 2008 until the 1551st day (18 February 2009) should be considered to have had its grounds. Despite the downward trend, the considerable volatility of the exchange rate allowed for the conclusion that the situation which occurred could hardly be deemed as unexpected.

Another application of the model was carried out while taking into account the first 2096 observations. This is the period preceding the second significant maximum. On 8 April 2011 the exchange rate was 3,0097. The mean value estimated on the basis of the first 2096 observations was equal to $\hat{m}(2096)=2{,}6631$, with a standard deviation of $s(2096)=0{,}2998$. In this case the attempt is to evaluate the probability of the event referring to the 2182 day, where the exchange rate reached the level of 3,9562. The estimation was thus as follows:

$$P(a2182(M(2182)-b2182)\leq x),$$

where it was calculated that $x=a2182(3{,}9562-\hat{m}s-b2182)=6{,}1068$ and $\exp(-e-x)=0{,}9978$.

This means that the probability, apriori calculated, that the maximum exchange rate will reach the level of 3,9562 is equal to 0,0022. The probability that an opposite event will occur is of course big, yet it cannot be the cause for accepting the model. What is interesting here is that the probabilities for the exchange rates which are close to the rates at the level of 3,9562 have now some practical relevance. For example:

$$P(\max_{t\leq2182}X(t)\geq3{,}6383)=1-0{,}87=0{,}13$$

$$P(\max_{t\leq2182}X(t)\geq3{,}7147)=1-0{,}95=0{,}05$$

or
$$P(\max_{t\leq2182}X(t)\geq3{,}7220)=1-0{,}953=0{,}047$$

Such exchange rates were reached on the days neighbouring the 2182nd day.
The third time the model was used to assess the probability of an event which was commonly thought as unpredictable. We refer to the incidence from January 2015. On 15 January 2015 the Swiss National Bank decided not to intervene in order to weaken the Swiss franc against the Euro. On that day the currency pair CHF/PLN exceeded even the level of 5 PLN on the Forex Exchange. In this paper, however, the NBP's exchange rates are the basis, since they provide the basis for calculating the loans denominated in Swiss francs. On 15 January 2015 (the 3043rd day) the NBP's exchange rate was 4,1611, reaching on the following days (23 January 2015 – the 3049 day) even the level of 4,3223. In this case (we are dealing with a currency sudden hike) we selected a period directly preceding the event under discussion. The

3042nd session was used as the basis for creating the model. On that day, the exchange rate of CHF to PLN was 3,5712 and hardly anything could be seen as boding a sudden change.

Not unlike in the two previous cases, the mean value and standard deviation were estimated, with the following result: $\hat{m}(3042){=}2{,}9066$ and $s(3042){=}0{,}4441$.

It is important to note that there is a clear increase of both of the values which were estimated. It favours extreme events.

For 15 January 2015 (exchange rate equal to 4,1611), the calculation was as follows:

$a_{3043}{=}4{,}0051$ and $b_{3043}{=}2{,}8535$ and and the distribution function $F({-}0{,}1154){=}0{,}3255$.

While for the 3049th day (23 January 2015 r.), the calculation was as follows: $a_{3049}{=}4{,}0056$ and $b_{3049}{=}2{,}8541$ and

$$x = a_{3049}\left(\frac{4{,}3223-2{,}9066}{0{,}4441} - b_{3049}\right) = 1{,}3362$$

As next, the value of the distribution function was calculated at this point: $F(1{,}3362){=}0{,}7689$.

This means that

$$P(\max_{t\leq 3043} X(t)\geq 4{,}1611){=}1{-}0{,}3255{=}0{,}7745$$

and

$$P(\max_{t\leq 3049} X(t)\geq 4{,}3223){=}1{-}0{,}7689{=}0{,}2311$$

We can notice that both events had reasonably big probabilities. The assessment of the probability that the situation will occur was possible owing to the significant volatility following the second (the 2182nd day) non-gradual change of the exchange rate. The change that occurred on that day could not be described using the proposed model. This is because until that time the CHF/PLN currency pair (disregarding the speculation period of the year 2008) had been a stable pair and low exchange rates and low volatility

had not allowed for taking advantage of the benefits involved in the modeling which uses extreme statistics.

Other Models

One of the models in which the assumption of stationariness can be overridden is the model based on Horowitz's Theorem (Horowitz, 1980).

Theorem 2. (Horowitz)

Consider $M_n{=}\max(\zeta_1,\zeta_2,...,\zeta_n)$, where ζ_i has a normal distribution and $\zeta_i{=}\xi_i{+}m_i$, where $E(\xi_i){=}0, Var(\xi_i){=}1$ and $cov(\xi_i,\xi_j){=}r_{ij}$. Let us also assume that $|r_{ij}|{<}\rho_{|i-j|}$, and $\rho_n \ln n{\rightarrow}0$. Let $\beta_n{=}\max\limits_{1\leq i\leq n}|m_i|{=}o((\ln n)^{1/2})$, then:

$$P\{a_n(M_n{-}b_n{-}m_n^*)\leq x\}{\rightarrow}\exp(-e^{-x}),$$

where constants m_n^* satisfy the condition:

$$\tfrac{1}{n}\Sigma_{i=1}^{n}\exp(a_n^*(mi{-}m_n^*){-}\tfrac{1}{2}\,(mi{-}m_n^*)^2){\rightarrow}1.$$

In this theorem, the constants m_n^* can determine the intervals of non-stationariness. The last equation from theorem 1 should be solved numerically finding constants m_n^* (equating the left side of the equation to zero). The fit of this model remains far from satisfactory. While maintaining determinations from point 3, we obtained:

$$P\{a_{1551}(M(1551){-}b_{1551}{-}m^*{}_{1551})$$
$$\leq{-}7{,}4780\}{=}0{,}999435$$

Number -7,4780 is the realisation of the left side of the above equation. In terms of the exchange rates of the CHF/PLN currency pair, this means that:

$$P(\max_{t\leq 1551} X(t)\geq 3{,}3167){=}0{,}000565$$

In this case it is not easy to consider the model to be truly warning against a significant change of the exchange rate. The values of the distribution

function of maximum are overestimated as the result of the use of the constants m_n^*.

This is likely to be the result of changing the levels without changing the volatility. It appears that it is likely that the volatility increase is in some cases sufficient. Suspecting that the distribution of a variable with sudden non-gradual changes may be a distribution with fat tails, Pareto distribution was used for the analysis. Reiss and Thomas (Reiss, Thomas, 2007) suggest applying this model for the analysis of similar phenomena. The distribution function has the following form:

$$F(x) = 1 - kx^{-\alpha}$$

for $x \geq k^{\frac{1}{\alpha}}$ $(\alpha > 0, k > 0)$.

This distribution function belongs to the domain of attraction of the type II distribution, that is:

$$F_{II}(x) = \begin{cases} \exp(-x^{-\alpha}) & for \quad x > 0 \\ 0 & for \quad x \leq 0 \end{cases}$$

In this case, however, the estimation itself of parameters α and k using the method of least squares proved to be unsatisfactory.

Conclusion

Extreme statistics distributions proved to be useful in the assessment of the probabilities of events which other methods find difficult to describe. The fact that the results are not always satisfactory is caused by the very nature of the phenomenon. In the currency market, and particularly as regards the CHF/PLN currency pair, we face rare phenomena determined by the Swiss National Bank's decisions. The stochastic models can reflect those phenomena only approximately. On the other hand, the application of causal models is not possible for we have no information on the bank's or possibly other entities' decisions. Perhaps such models could show trends, but because of the equal weight usually ascribed to individual moments in time in the estimation process, a point prediction of sudden changes seems impossible, hence the proposal of estimation $\max_{t \leq n} X(t)$. "Predictions" of this type do not define the value of variable $X(t)$ at a specific moment in time precisely, yet they specify the distribution of the maximum value which is especially significant for the issue under discussion.

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Statystyki ekstremalnew analizie zmienności kursu walutowego CHF/PLN

Abstrakt

W artykule przedstawiono analizę zmienności kursu wymiennego franka szwajcarskiego do złotego (CHF/PLN). Problem ten ma ważne znaczenie praktyczne. Z uwagi na znaczną zmienność, kurs tej pary walutowej istotnie wpływa na sytuację kilkuset tysięcy kredytobiorców. W pierwszym dziesięcioleciu obecnego stulecia udzielono wielu kredytów (głównie mieszkaniowych) nominowanych we frankach szwajcarskich. Powoduje to często poważne trudności w obsłudze długu i może mieć poważne konsekwencje społeczne i gospodarcze. Dlatego też rządzący starają się rozwiązać ten problem na różne sposoby, chociaż w chwili składania artykułu do druku żadne rozwiązanie nie zostało wdrożone.

Słowa kluczowe: frank szwajcarski, złoty polski, statystyki pozycyjne, obszar przyciągania, grube ogony, rozkład maksimum