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Double Perspective DEA on appraising property's fair market value by Brazilian Standards

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Abstract:

Objective: This article innovates when we incorporate the statistical analysis to the method of Double-Perspective Data Envelopment Analysis (DP-DEA), with the objective of obtaining an estimate of greater accuracy and reliability according to the assumptions of the Best Unbiased Estimator (BUE).

Design / Research Method: Double Perspective Data Envelopment Analysis (DP-DEA) is an extension to Classical Data Envelopment Analysis (DEA) for estimating efficiency, asset values, indicators, and other attributes from two perspectives, achieving a common result is the main objective. This article innovates in DEA methodology, in two aspects: 1. To demonstrate the ability of the DP-DEA to perform at intervals the estimation of values from a random sample; 2. Through the statistical analysis, making estimates of central tendency according to the assumptions of the Best Unbiased Estimator (BUE – Best Unbiased Estimator).

Conclusions / findings: The practical procedures performed step by step through DP-DEA according to the assumptions of the BUE, presented in its main findings and conclusions are: 1. Incorporation of statistical analysis to the DP-DEA method, which assumes assumptions of properties of the Best Estimator Non-biased; 2. Within the scope of the DEA, it presents an innovative capacity to make estimates from random samples, and; 3. At the end of the article, by simulation, able to validate modeling through the variation of property characteristics, demonstrating that the estimation of the corresponding values is consistent according to the market's expectations.

Originality / value of the method: This article opens new avenues to be explored by the DEA community. Firstly, as a tool for valuing assets, according to the Comparative Market Data Method. In

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Brazil, DP-DEA has been approved by the Brazilian Association of Technical Standards for this purpose. Another innovation is to evaluate performance which results from common gain according to two perspectives, which interact in the process or procedure under analysis.

Key words: Double Perspective Data Envelopment Analysis; Real Estate Appraisal; Best Unbiased Estimator

JEL: C61, D12

1. Introduction

This article presents the evolution of a new field of application and research for Data Envelopment Analysis (DEA) through an approach in valuing assets.

The methodology of Double-Perspective Data Envelopment (DP-DEA) (Novaes 2002; Lins et al. 2005) was developed according to the classical theory of Data Envelopment Analysis (Charnes et al. 1978; Banker et al. 1984), which is ground breaking in the area of DEA methodology due to the ability to make estimates from a set of data (Sampaio 2010). Thesis reports an applying of Double Perspective Data Envelopment Analysis to the Benchmarking Process of Public Utilities.

This article applies the DP-DEA to estimate the fair market value of property under the Direct Comparative Method of Market Data (Lewis 1999). Similarly, to regression analysis, DP-DEA studies the dependence of one or more variables explained by one or more explanatory and / or independent variables, to estimate a range of viable values from two different perspectives and to find common satisfactory results.

The case in question, the objective is to estimate the fair market value in which a property can be traded, that is, the central tendency value where the supply and demand of the commodity – property – serves the interests of the buyer and the seller. The plausibility of this assertion is best understood according to the law of supply and demand (Whelan, 1996), according to where there is a region of central tendency, where the supply and demand curves meet, and the fair market value is established (Misra, 2013).

The main innovation presented in this article is to demonstrate the use of statistical fundamentals, applied to the DP-DEA, has the property of establishing the best unbiased estimator.

In section 5, we verify, instead of establishing a single value estimate, a range of admissible values delimited by the enclosures that encapsulate the analyzed data set is defined. The hypothesis formulated is that the envelopes delimit the viable region of occurrence, which satisfy the laws of supply and demand (Shephard 1975) applied to the real estate market, according to the buyer and seller's views.

To summarize, we contextualize the main contributions of DP-DEA in the research and the application which is to be portrayed in this article, through the following themes:

- Contextualization of the theoretical innovation of the DP-DEA approach in relation to the DEA methodology.
- Origin, history and motivation of the DP-DEA (Double Perspective DEA) methodology (Novaes, 2002).
- Fundamentals and mathematical formulation of the DP-DEA method.
- Criteria and assumptions adopted to establish the best DP-DEA estimator.
- Formulation and characterization of the equation of the central tendency estimator.
- History and relevance of Real Estate Evaluation Standards in Brazil
- Regulatory and Market Context for applying the methodology in Brazil.
- Application of DP-DEA according based on the real estate market data.

2. Contextualization of the theoretical innovation of the DP-DEA approach in relation to the DEA methodology

The theoretical context of the scope of the DEA methodology is portrayed in an article by Ali Emrouznejad and Guo-liang Yang (2017), when researching the diversity of its application in various areas of human knowledge in the last four decades. We verify that among approximately fifty related keywords like the mostly used, they are not cited in the focus of this article: Real Estate Appraisal; Direct Market Data Method, and; Best Unbiased Estimator.

The originality and innovation introduced by DP-DEA corresponds to its ability to estimate values for one or more dependent variables through one or more explanatory and / or independent variables, introducing a new area of research and application for the DEA methodology.

According to the most recent bibliographies, one of the main obstacles to the greater diffusion of the DEA is the analysis of productivity and efficiency which lies in the fact that it has no statistical basis (Ray 2004). The same authors assume that two different samples, which use the same input packages and produce sets of different quantities of products, would lead to different measures of the production frontier.

Bayraktar et al. (2008) also note that the sources of uncertainty are in the process of matching demand with supply. In this application, by encapsulating observed market data, uncertainty translates into the variability of occurrences which can satisfy each perspective of demand and supply, a condition that will be better explained in the 5 and 6 section.

It is important to verify what the DEA community reaction, has been when introducing the use of a more statistical basis, as we can verify in the following publications:

- Wanke et al. (2017) describe several new parametric models to deal with input/output uncertainty in DEA;
- Banker's (1993) interpretation of the DEA as MLK – Maximum Likelihood Estimation, uses the F statistic to establish greater reliability to the adopted model;
- Land et al. (1993) proposed a DEA approach with probability restrictions;
- Varian (1985) proposed a statistical test of data consistency with the WACM – Wax Costs Minimization Axiom and WAPM – Weak Axium and maximum profit;
- Simar and Wilson (1998) use consistent bootstrap techniques to generate empirical distributions of efficiency outcomes;
- Subhash (2004) notes that a new way to deal with input/output uncertainty in DEA depends on the randomness of probability distributions;
- Morita and Seiford (1999), Brázdik (2004) and El-Demerdash et al. (2013) observe that a restriction limits the use of the stochastic DEA model in cases where the event is unique or deterministic, a priori or posteriori of

probability distributions needs to be addressed. However, the uncertainty of input/output may be related to the inaccuracy and not to the randomness. Where, some Fuzzy DEA methodologies are compared.

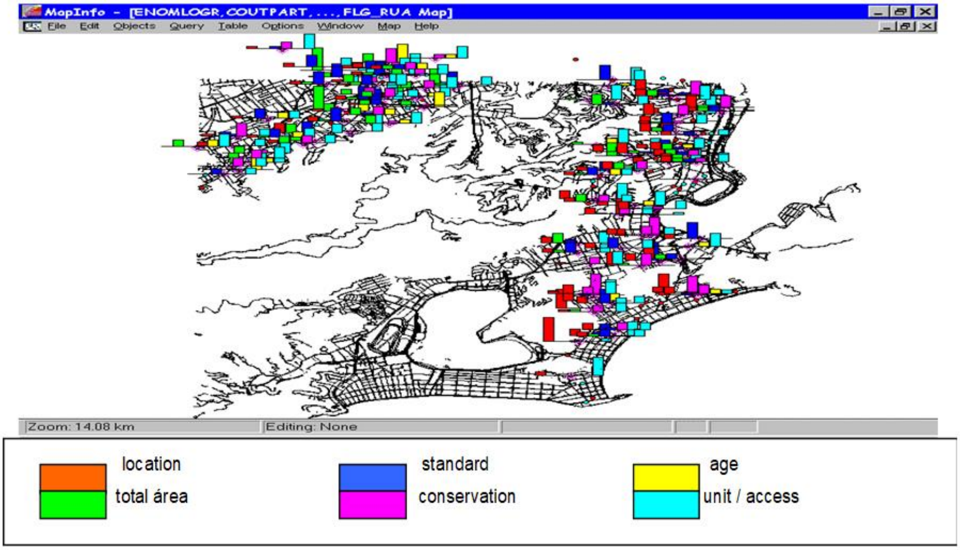
- Bayraktar et al. (2008) investigate the impact of value range of practices on retailer performance and makes the economic premise that uncertainty is generally defined as unknown future events which cannot be predicted quantitatively within useful limits, thus making the occurrence of uncertainty unpredictable.

3. Origin, history and motivation of the Double Perspective DEA

The Double Perspective DEA technique was developed within the doctorate thesis of Luiz Fernando de Lyra Novaes (Novaes L.F., 2002), under the guidance of professor Marcos Pereira Estellita Lins. The introductory international publication of this method on a paper of Lins et al. (2005) "Real Estate Appraisal. A Double Perspective Data Envelopment Analysis Approach".

A first important result was the presentation of a conference paper, awarded first prize in the III Symposium of Appraisal Engineering – Avaliar 2002 (see Novaes et al., 2002), promoted by the Brazilian Association of Institutions for Financing Development (ABDE). One relevant approach presented in that occasion, was the capacity of the parameters forming of frontier hyper plane defined by DP-DEA Multipliers Model (Novaes 2002) on interpreting the real estate buyer's behavior, similar with the price-elasticity, verified on lower surface. Each real estate price-elasticity graph bar is geocoded, e.g., see Figure 1.

Figure 1. GIS representation of physical features on apartments assessment



Source: Novaes (2002).

Over the map of Rio de Janeiro municipal district was geocoded factors graph bars that represents the main factors trade-off, that explain the buyer interest for each apartment analyzed, on response of what are the relative importance of real estate valuation factors, based on sales data compiled by a real estate market.

When you are looking for a property, new home or starting a home renovation, it is important that you understand the factors influencing the property valuation.

Location: The axiom "location, location, location" emphasize the importance of this factor on chose a property, and the reason it is such a prevalent cliché is because there is truth behind it. Proximity to beaches, laser places, employment centers, medical facilities, shops, and schools is a determining factor for many families and young couples when buying a home. Proximity to a wide array of local amenities and good transport links increases the value of your potential property.

Age and Conservation: The age of a property doesn't automatically reduce its value. However, the condition of conservation of a property makes a huge difference. An old but well-maintained property can achieve a valuation that is the same as that of a new build of equivalent specifications; sometimes, even higher.

Area and number of rooms: The size of an apartment on combination with the number of rooms affects the value of the property. For example, increasing the number of bedrooms and bathrooms is a good move to increase the value of your home, whereas removing facilities or reducing the number of bedrooms is not.

Standard: Represents the quality of accessories and materials on wall-covering, floorcovering, facades, common area and apartment. A building class distinguished by a quality rating of the building construction, varying from luxury until minimum recovering. The standard reflects directly on value real estate variation.

Unit/access: The condition of apartment access distinguished mobility and how is increased on exclusivity on the use of the building amenities, service and social hall and lift usage. These conditions are correlated with the investment made on the construction of the building, representing what is offered on comfort to residents.

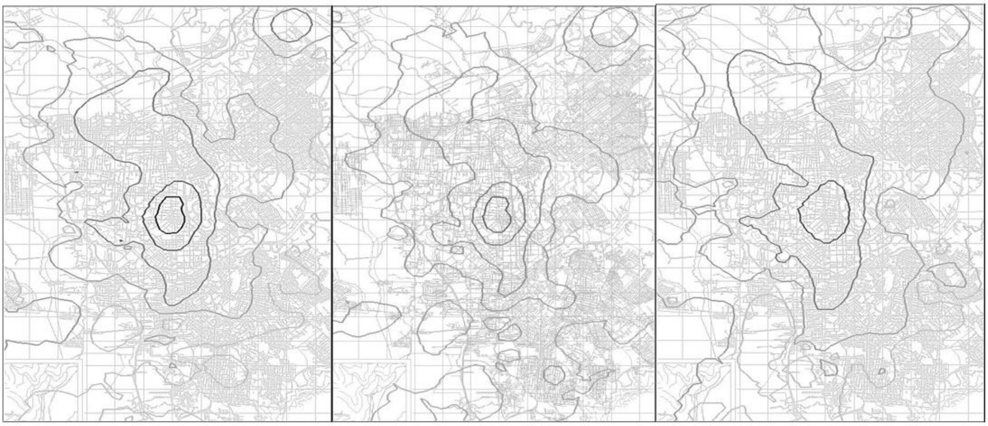
The influence of each real estate factor price elasticity is distinguished by the graph bar geocoded exactly on the location of each apartment. The red bar represents the location, the blue represents more privacy, yellow is apartment age, dark blue the building standard, purple is conservation and green are apartment dimension. Observing the map, we can see a buyer behavior varying with its location. Some factor is more important as if formed by a location cluster, permitting interprets the buyer's behavior changing in function of the apartment localization into the map. On the cluster formed by coastal apartments, we can conclude that the location is prevalence. However, for the cluster formed by suburban region apartments other factors, total area, conservation and units/access are preponderant and they do not account with none influence of location.

In 2009/2011 it was performed NBR 14.653 review – Brazilian National Uniform Standards of Professional Appraisal Practice (ABNT 2011). The DP-DEA – Double Perspective Data Envelopment Analysis method (Novaes 2002) was included within the scope of quantitative methods recommended for appraisal.

On page 17 the Standard recommends: "Other analytical tools for induction of market behavior, considered of interest by the appraisal engineer, such as space regression, data envelopment analysis and artificial neural networks, may be applied, given that theoretical and practical point of view where duly justified, with the inclusion of validation, where relevant. In the case of the use of data envelopment analysis (Double Perspective-DEA), Appendix D must be observed".

Novaes and Paiva (2010 lining up with Appendix D ABNT 2011), established an approach to solve the LOOP – Law of One Price arbitrage (Eckard 2004). A general equilibrium model of real estate values developed to analyze property price per square meters variation over a digital map of the urban area of the city of Joinville compared with MRA – Multiple Regression Analysis. All computational results were geocoded on a GIS – Geographic Information System generating scales of price isolines over the Map, enabling compare accuracy between DP-DEA and MRA. Comparing Figure 2 with Figure 3 and Figure 4 with Figure 3, is perceptively the fitness adjustments of DP-DEA value isolines, e.g., Figure 2, with the value observed isolines than MRA isolines.

Figure 2. DP-DEA Figure 3. Observed data Figure 4. MRA



Source: Novaes, Paiva (2010: 4-6).

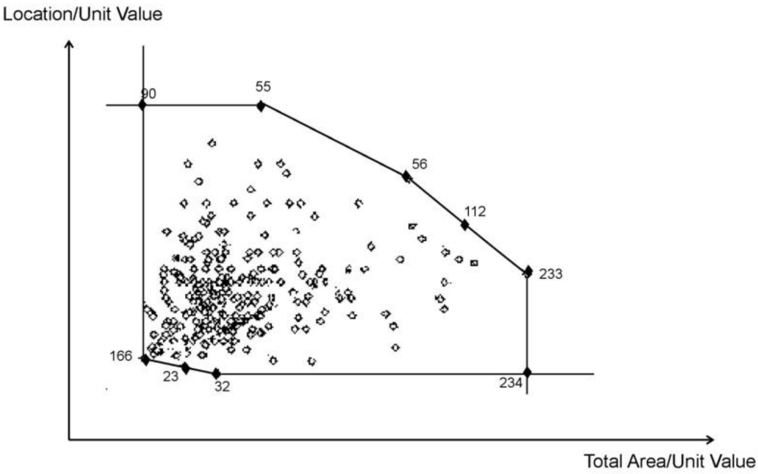
4. Fundamentals and mathematical formulation of the DP-DEA method

DP-DEA Seller Perspective Surface is stablished by the efficient price on seller behavior, that wishes to maximize the output (selling value), given the input (property attributes). Symmetrically opposite, DP-DEA Buyer Perspective Surface is established by the efficient price on buyer behavior, that wishes to minimize its input (purchase value), given the outputs (property attributes). The problem is treated through DEA linear programming problems, which provide two DEA frontiers: the seller and the buyer, therefore the name DP-DEA is given to represent the behavior interaction by seller and buyer on realizing a market transaction.

In this paper, the DP-DEA method is applied to a dataset consisting of properties market value of real estate transactions which occurred in some specific region. A panel data, representing the scope of an observed real estate market, contains the possible transaction values influenced by the supply or demand as a function of properties characteristics. Modeling starts by selecting" observed real estate data, with "m" independent variables, considering the usual main factors that report the characteristics property attributions and "s" dependent variable, mainly considering the property value and/or sale velocity, real estate build quality and/or other output of interest. The DP-DEA model determines a subset composed of "k" data which belongs to each perspective's enveloping surface. This data is considered that the best attends either one or the other perspective and defines the segments of the enveloping surface, thus motivating the envelope form of DEA CCR or BCC models (see Lins, Meza 2006). The contained subset, not belonging to this surface, is formed by the remaining "n – k" inefficient data for each perspective. The computation of the normalized distance of each observed unit requires the solution of a linear programming problem (Lins et al. 2005: 5-14).

Figure 5 shows the area contained under the enveloping surfaces, which correspond to a panel data of real estate market. It results from the intersection of the set of supply possibilities and the set of demand possibilities (see Färe et al. 1996; Shephard 1970). The buyer's perspective frontier defined by units 166, 23, 32 and 234; and from the seller's perspective frontier defined by units 90, 55, 56, 112 and 233. Regression literature exposes that the desirable statistical properties fall into two categories: small-sample or large-sample. Under DP-DEA assessment approach, fundamentally, we observe the assumptions described and the following considerations.

Figure 5. DP-DEA Envelopment Surfaces – Seller and Buyer Perspectives



4.1 For large-sample

Normal Approximation Rule states, as when dataset increase, the distribution of the sample mean $\bar{X}$ concentrates more and more around its target the population μ (Wonnacott, Wonnacott, 1990). This rule known how Central Limit Theorem can be achieved to any PDF Probability Density Function (Davidson, 2008).

4.2 For small-samples

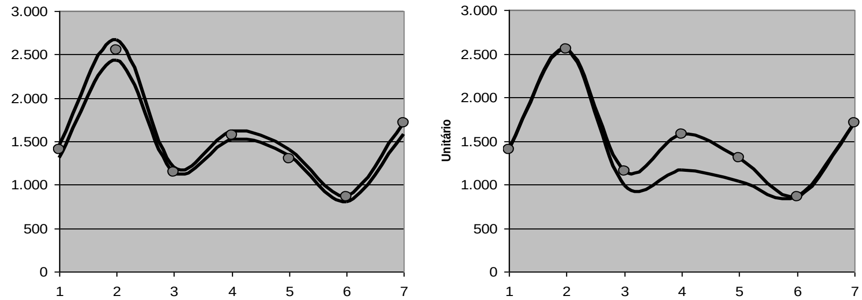
Comparing the envelopes DP-DEA results with confidence interval of multiple linear regression, we can examine the differences of these two techniques. First applying (Souza 2009: 5-15) compared the L.R. – Linear Regression Confidence Interval at 80% with the Frontiers defined by DP-DEA with database extracts of different sources and observes these results. At a data set formed with 2 variables and 7 data (Zeni 1990), e.g. see Table 1 and Figure 6. The second applying, with 2 variables and 35 data (Lins et al. 2003). Although the surfaces of the envelopes confidence interval data are best fit, it is found 22 observed data are outside, see Table 2. However, all data observed are contained by the envelopes DP-DEA, see Figure 7 and Table 2.

Table 1. Sample composed by 2 variables and 7 data comparing L.R. with DP-DEA estimate

	Data		Regression 80% Results				
Unit (R$/m2)			Infered				
Observed	Dts.(m)	Area(m²)	($/m²)	Lower	Uper	Buyer Pp	Seller Pp
1,400.00	1,800.00	2,500.00	1,381.04	1,316.52	1,448.72	1,400.00	1,400.00
2,550.00	450.00	2,800.00	2,547.29	2,427.91	2,672.54	2,550.00	2,550.00
1,150.00	1,350.00	7,700.00	1,176.80	1,145.14	1,209.34	985.61	1,150.00
1,575.00	500.00	11,000.00	1,572.55	1,528.07	1,618.33	1,163.53	1,575.00
1,300.00	720.00	12,000.00	1,373.84	1,337.69	1,410.97	1,034.06	1,300.00
860.00	1,600.00	14,700.00	840.16	804.25	877.68	860.00	860.00
1,710.00	200.06	14,800.00	1,645.10	1,583.20	1,709.41	1,710.00	1,710.00

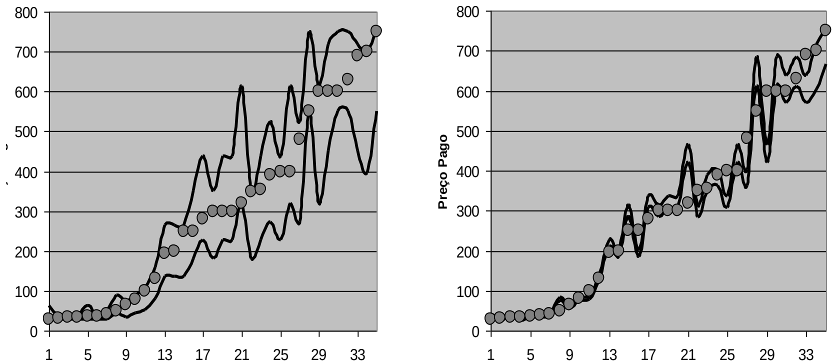
Source: Zeni (1990: 9-16).

Figure 6. Comparing L.R. Confidence Interval (80%) fitness with DP-DEA Buyer and Seller Surfaces



Source: Zeni (1990: 9-16).

Figure 7. Comparing DP-DEA Buyer and Seller Surfaces fitness with L.R. Confidence Interval (80%)



Source: Souza (2009).

Table 2. Sample composed by 3 variables and 35 data comparing L.R. with DP-DEA estimate

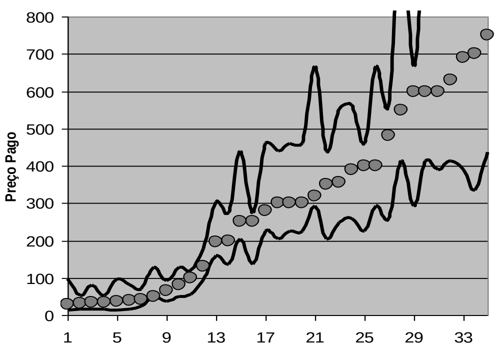
Data set			Regression 80% Results			DP-DEA Results	
Price Observ. (U$ millions)	**Return(%)**	**Risk (%)**	**Inferred (U$/m²)**	**Inferior**	**Superior**	**Inferior**	**Superior**
30.00	70.00	50.00	37.07	32.79	41.91	30.00	64.13
32.00	40.00	10.00	29.45	27.30	31.76	30.00	38.00
33.00	40.00	5.00	36.69	33.18	40.57	30.00	38.00
35.00	40.00	10.00	29.45	27.30	31.76	30.00	38.00
36.00	70.00	50.00	37.07	32.79	41.91	30.00	64.13
38.00	40.00	5.00	36.69	33.18	40.57	30.00	38.00
42.00	50.00	10.00	39.57	36.82	42.52	30.00	46.71
50.00	100.00	20.00	79.50	74.90	84.39	43.81	90.26
65.00	80.00	20.00	59.17	55.92	62.60	34.60	72.84
80.00	100.00	20.00	79.50	74.90	84.39	43.81	90.26
100.00	120.00	40.00	81.23	77.21	85.45	53.02	107.68
130.00	180.00	50.00	129.45	124.33	134.79	80.63	159.95
195.00	300.00	80.00	219.32	210.47	228.54	135.87	264.47
200.00	300.00	120.00	192.84	184.42	201.66	135.87	264.47
250.00	400.00	100.00	299.05	284.92	313.89	181.90	351.58
250.00	300.00	120.00	192.84	184.42	201.66	135.87	264.47
280.00	500.00	200.00	322.51	308.22	337.47	227.94	438.68
300.00	400.00	100.00	299.05	284.92	313.89	181.90	351.58
300.00	500.00	200.00	322.51	308.22	337.47	227.94	438.68
300.00	500.00	200.00	322.51	308.22	337.47	227.94	438.68
320.00	700.00	300.00	442.74	420.30	466.38	320.00	612.89
350.00	400.00	100.00	299.05	284.92	313.89	181.90	351.58
355.00	500.00	130.00	369.75	351.58	388.86	227.94	438.68
390.00	600.00	250.00	382.50	364.26	401.66	273.97	525.79
400.00	500.00	200.00	322.51	308.22	337.47	227.94	438.68
400.00	700.00	300.00	442.74	420.30	466.38	320.00	612.89
480.00	600.00	250.00	382.50	364.26	401.66	273.97	525.79
550.00	1,000.00	400.00	648.04	612.03	686.16	550.00	750.00
600.00	700.00	300.00	442.74	420.30	466.38	320.00	612.89
600.00	1,000.00	400.00	648.04	612.03	686.16	550.00	750.00
600.00	870.00	280.00	603.49	571.15	637.67	450.33	717.50
630.00	1,000.00	400.00	648.04	612.03	686.16	550.00	750.00
690.00	870.00	280.00	603.49	571.15	637.67	450.33	717.50
700.00	800.00	150.00	658.33	604.05	717.48	396.67	700.00
750.00	1,000.00	300.00	709.97	667.71	754.90	550.00	750.00

Source: Lins et al. (2003: 1413-1424).

The graphs are classified by the observation value (graph plotted points) rather than the inferred value (since this is not necessarily within the confidence), e.g. see Table 2.

At first glance RL takes "advantage". However, 22 of the 35 RL points are outside the 80% confidence interval, see Table 2. To place them within the range, we change the significance successively until their total inclusion. The new chart, indicating the significance used, see Figure 8. The RL has its significance close to 0.0%, which in statistical terms corresponds to a deterministic interval. The RL range is broader than DP-DEA.

Figure 8. L.R. Confidence Interval ~100%

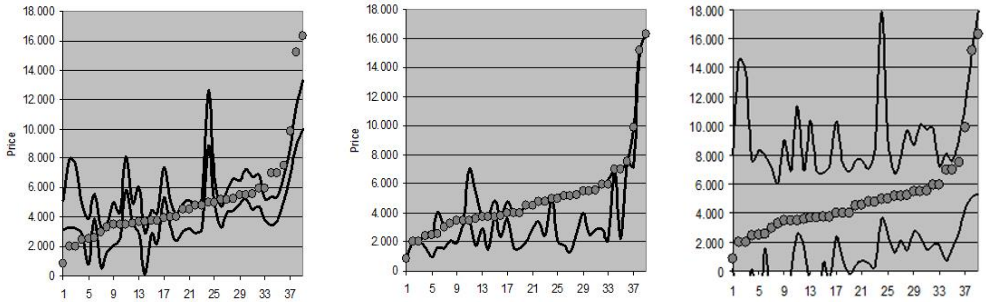


Source: Souza (2009).

The third base is taken from Maddala (2001). Table 4.7 – Price of Rural Property Land, 5 (five) independent variables in 39 observations. They refer to rural land prices near Saratoga, Florida with the following variables: Price (dependent US $,acre), WL (bare ground), DA (distance to airport miles), D75 (distance to highway miles), A (acreage), MO (month of actual transaction).

It is observed that the regression does not contain all the points observed in its 80% CI graph, especially in the extremes. By changing the significance successively until their total inclusion, we have the new graphic profile with C.I. approximately equal 100%, which in a quick examination gives more fitness to DP-DEA, see Figure 9.

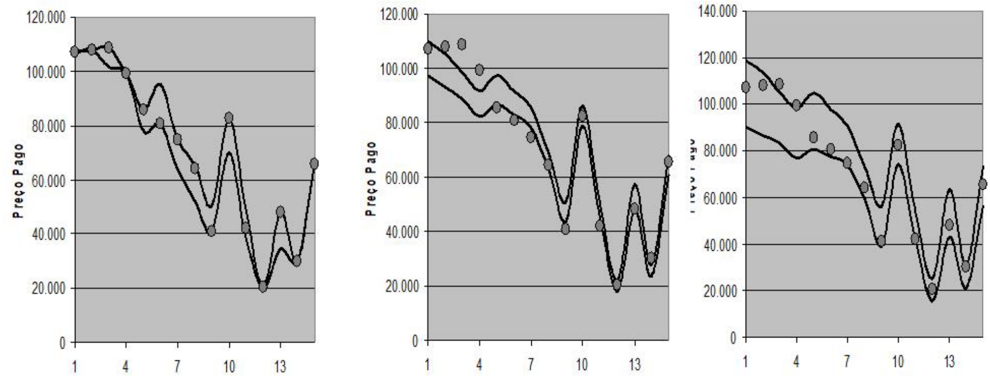
Figure 9. Comparing L.R. C.I. 80% with DP-DEA Envelopment and L.R. C.I. 100%



Source: Souza (2009).

The fourth example is provided by the "Manual of Econometrics" (Vasconcellos (1999)), e.g., see Figure 10. This is an exercise in section 6 of the section of "Multicollinearity". Visual comparison of the graphs shows that the shape of the curves is quite similar, however the regression does not contain several points observed within the confidence limits. For this to occur we must return to the CI of the model data and make its uncertainty equal to 1%. The new chart comparative is as above, with a slight visual disadvantage for the regression that still has a point outside (only included if the significance is 0.20%).

Figure 10. Comparing DP-DEA. with L.R. C.I 80% and 100%

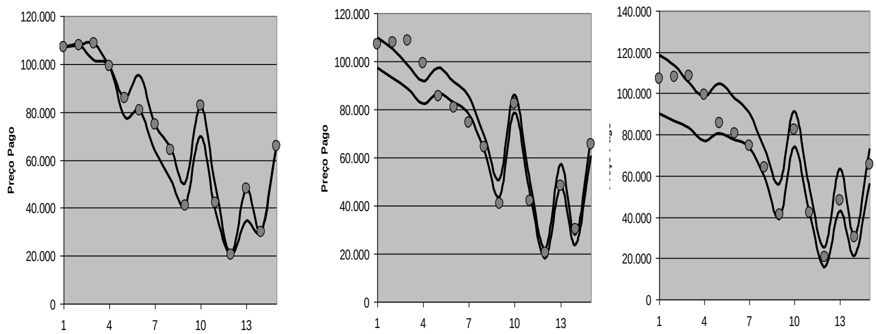


Source: Souza (2009).

The fifth and final example comes from the National Institute of Standards and Technology (NIST) data collection of the Department of Commerce, Technology Administration of the United States. This body provides certified results for regression analysis of varied series, with an increasing degree of difficulty.

The goal is to report these results to companies and developers to compare with those produced by commercial software. The series we will use as an example is Longley. This classic statistical series of work in America was one of the first used to test the computational precision of the minimum square. The response variable (y) is the total derivative employment and the prediction variables are implicit GNP deflator with 1954 = 100 (x1), gross national product (x2), unemployment (x3), size of armed forces (x4), the illegal population above the age of 14 (x5) and the year (x6). Three points are not contained in IC = 80.0% of RL. For this to occur we must have CI = 97.5%, see Figure 11.

Figure 11. Example 5 Data Base 6x16



Source: Souza (2009).

The DP-DEA is an iterative non-linear process, e.g. see a practical demonstration on section 9. It seems to be an advantage of the DEA to provide an immediate critique of data: if a given point extends the boundaries too much, it can be removed from the sample and the process restarted. New frontiers are generated, thus successive frontier modeling. The examples presented here indicate that highly collinear models are better explained by RL, whereas in small samples with almost no data, DEA is a very valid alternative.

5. Criteria and assumptions adopted to establish the best DP-DEA estimator.

Instead of obtaining only a single estimate of $\widehat{\boldsymbol{\theta}_j}$, $j = 1, \dots, n$, DP-DEA obtain three estimates of $\widehat{\boldsymbol{\theta}_j}$. Two estimators are the buyer efficient value and the seller efficient value, i.e., the independents variables "xij", with i = 1 to m , explained by dependents variables "yrj", with r = 1 to s, representing respectively by projections $\widehat{\boldsymbol{\theta}}_{j\ Buyer} = (\widehat{\boldsymbol{x}}_{ij}, \widehat{\boldsymbol{y}}_{rj})_{Buyer}$ *and* $\widehat{\boldsymbol{\theta}}_{j\ Seller} = (\widehat{\boldsymbol{x}}_{ij}, \widehat{\boldsymbol{y}}_{rj})_{Seller}$. The key concept underlying interval estimation is formal defined by DP-DEA approach (see Novaes 2002; Lins et al. 2005).

The interval between $\widehat{\boldsymbol{\theta}}_{j\ Buyer}$ and $\widehat{\boldsymbol{\theta}}_{j\ Seller}$ are established by buyer and seller envelope surfaces, e.g. see Figure 12. Thus, interval estimation, in contrast with point estimation, provides a range of possible values within the third estimator $\widehat{\boldsymbol{\theta}_j}$ lies, the central tendency value (Novaes, Paiva 2011).

According to which:

See respectively notation on next Table 3.

Observed data $\theta_{j\ (j=1,\dots,n)} = (x_{ij}, y_{rj})_{(i=1,\dots,m\ ;\ r=1,\dots,s)} \exists\ A\ |A$ is a dataset of a population observed.

The buyer and seller envelopes surfaces are performed respectively when $\theta_j = \widehat{\boldsymbol{\theta}}_{jBuyer}$ and $\theta_j = \widehat{\boldsymbol{\theta}}_{jSeller}$, j =1,…,n, respectively.

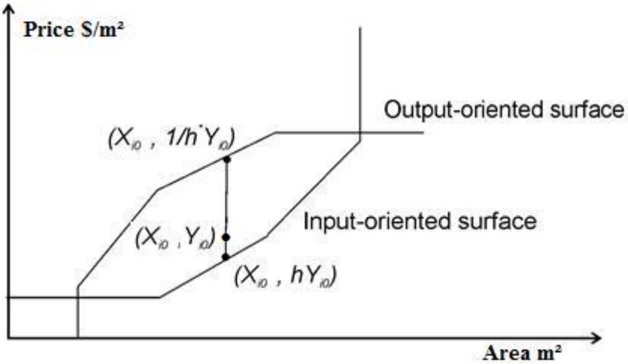
When θ_j matches the buyer or the seller envelopes surfaces, it has efficiency equal a 1 under each perspective, as like in Figure 12, respectively h_{Bj} =1 and 1,h_{Sj} = 1.

h_B and h_S are the decision variable estimated under DP-DEA approach (Novaes, Paiva 2011: 2-14).

$\widehat{\boldsymbol{y}}_{rjBuyer} = \boldsymbol{y}_{rj} \times \boldsymbol{h}_{Bj}$ and $\widehat{\boldsymbol{y}}_{rj\ Seller} = \boldsymbol{y}_{rj} \times \boldsymbol{h}_{Sj}$ (Novaes 2002; Lins et al. 2005).

Figure 12 shows the interval between $\widehat{\boldsymbol{\theta}}_{j\ Buyer}$ and $\widehat{\boldsymbol{\theta}}_{j\ Seller}$,the projections of $\theta_{j\ (j=1,\dots,n)} = (x_{ij}, y_{rj})$ on DP-DEA envelopes surfaces, i. e., the hyper-planes formed by maximization of outputs and the minimization of input on envelopment PPL form. The variable h and 1,h' correspond to efficiency determinate by DP-DEA approach, in such a way, which input under a buyer's perspective is the output under the seller's perspective and vice-versa (see Novaes 2002; Lins et al. 2005).

Figure 12. DP-DEA Buyer and Seller Perspective Surfaces



DP-DEA Buyer and Seller Perspective Surfaces are defined to Real States Appraisal application, in observing the Law of Supply and Demand. The law of supply and demand theory explains the interaction between the supply of a resource and the demand for that resource. The theory defines the effect that the availability of a particular product and the desire (or demand) for that product has on its price (Sholarin, Awange 2015).

6. Formulation and characterization of the equation of the central tendency estimator

The LOOP – Law of One Price (Eckard 2004) main principle states that when assets are identical in all aspects of value or characteristics, they must have the same price under market equilibrium. If two identical assets have different prices, there will exist an arbitrage opportunity and exploring this opportunity will help ensure that prices of the two assets converge.

An asset's fundamental value is the price that well-informed investors are willing to pay in a free and competitive market. By the Law of One Price, investors would assess values such that equivalent assets have the same price. There can be a temporary difference between the market price of an asset and its fundamental value. Likewise, security analysts make their living by researching the prospects of various firms and recommending which stocks to buy, because their price appears relatively low to the fundamental value, and which to sell, because their price seems relatively high to the fundamental value.

Table 3. Notation DP-DEA Central Tendency Estimator

$\hat{\theta}_j = (\hat{x}_{i\theta j}, \hat{y}_{i\theta j}) \quad - \quad \textit{Central Tendency Estimate}$

$$\sum_{j=1}^{n} (\hat{y}_{i\theta j} - y_{ij}) = 0 \qquad (1)$$

$$/ \quad \hat{y}_{i\theta j} = Z(y_{i\,Sj} - y_{i\,Bj}) + y_{i\,Bj} \qquad (2)$$

$$\hat{y}_{i\theta j} \geq 0 \qquad Z \geq 0 \qquad (3)$$

$$\textit{By Seller's Perspective} \qquad y_{i\,Sj} = {y_{ij}}/{h_{Sj}} \qquad (4)$$

$$\textit{By Buyer's Perspective} \qquad y_{i\,Bj} = y_{ij} \times h_{Bj} \qquad (5)$$

$$Z = \frac{\sum_{j=1}^{n} y_{ij} - \sum_{j-1}^{n} h_{Bj} \times y_{ij}}{\sum_{j=1}^{n} \left(\frac{1}{h_{Sj}} - h_{Bj} \right) \times y_{ij}} \qquad (6)$$

$$\textit{and} \qquad \hat{x}_{i\theta j} = x_{ij} \qquad (7)$$

Notation

$\hat{y}_{i\theta j}$ – j Central Tendency Estimate

$y_{i\,Sj}$ – Projectedvalue of data j on the enveloping surface on seller's perspective

$y_{i\,Bj}$ – Projectedvalue of data j on the enveloping surface on buyer's perspective

y_{ij} – Real estate j price observed

Z – escalate

h_{Sj} – Real estate assessed j deviations to the enveloping surface on seller's perspective

h_{Bj} – Real estate assessed j deviations to the enveloping surface on buyer's perspective

$\hat{x}_{i\theta j}$ – j Factors or Characteristics Estimate

x_{ij} – j Observed Data Factors or Characteristics

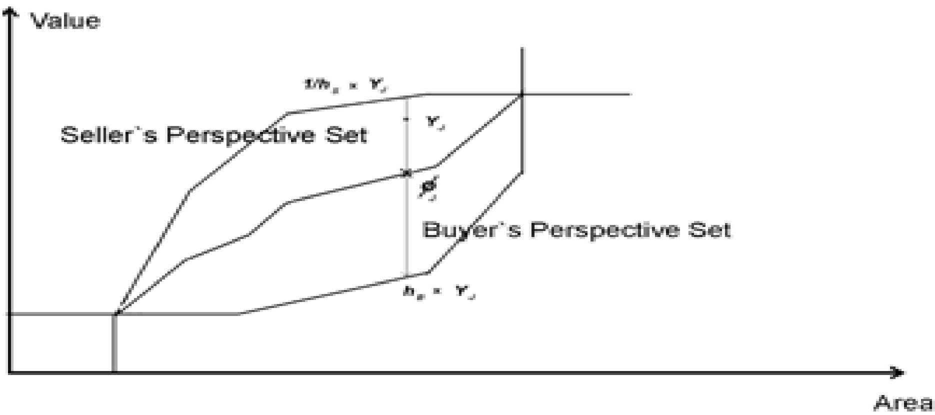
The real estate market assumptions states that no two distinct assets are identical in all aspects. There are always some differences of characteristics, maybe the localization, configuration or the size of the lot and the house or something else. The process of valuation requires that we find assets comparable to the one whose value we want to estimate and make judgments about which differences are important on their value to investors. This specific market equilibrium point is achievable when buyer and seller engage in a dispute and have attended their own interests of all kinds on the value of a specific real estate.

A strategy is introduced to determine central tendency estimates. The DP-DEA central tendency estimator between the two encapsulating surfaces (Novaes, Paiva 2010), is the dependent variables $\widehat{\theta_j}$ estimated by the minimization of the median absolute deviation of the whole distribution determined by the escalate Z (minimum-distance between the mass of dataset).

6.1. Central tendency estimates piece-wise line

In section 9, DP-DEA software estimates the dependent variables Φj by the minimization of the median absolute deviation of the whole distribution determined by one estimated escalate Z. (Novaes, Paiva 2010). The LOOP,DP-DEA piece-wise line on R2, e.g. Figure 13, applying all possible variations of Φj (Novaes, Paiva 2010).

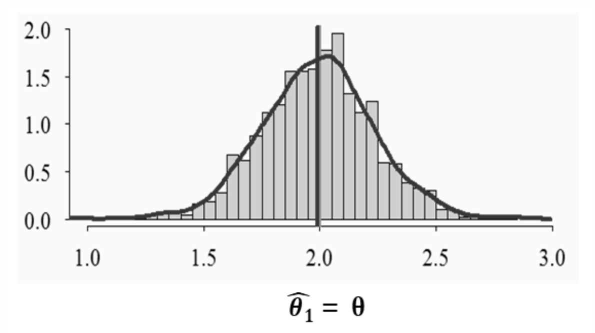
Figure 13. Central tendency piece-wise estimator line and the interval between Seller and Buyer Perspective Surfaces



6.2. Best Unbiased Estimator (BUE) Assumptions applied for DP-DEA modeling

In this section we describe how we deal with the DEA community's dilemma, already mentioned in item 2, when performing the DP-DEA model incorporating the statistical analysis with the objective of defining the best estimator of the central tendency value of the data evaluated.

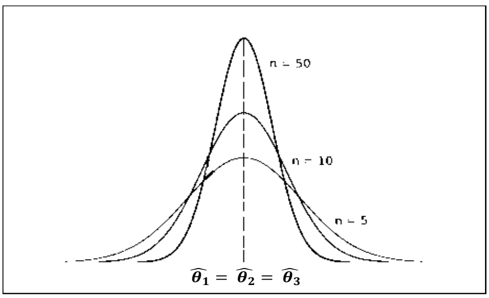
Figure 14. No tendency



In the practical modeling presented in section 9 of this article, the statistics used was to observe the three fundamental properties desirable for estimators, namely: non-bias, efficiency and consistency. The following summarizes the main aspects of each of these properties:

No Tendency: An estimator is said to be non-biased when its sample distribution has mean equal to the parameter to be estimated θ = Θ, e.g. Figure 14.

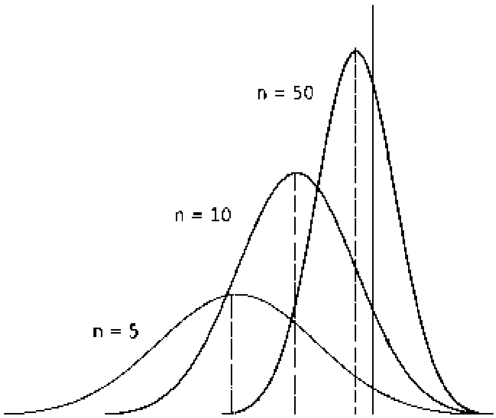
Figure 15. Efficiency



Efficiency: It is considered that between two non-biased estimators, the one with the smallest variance is called an efficient estimator.

That is, if Variance ((θ_1)) <Variance ((θ_2)) then (θ_1) is efficient, since (θ_1) and (θ_2) are non-biased estimators for Θ, e.g. Figure 15.

Figure 16. Consistency



Consistency: If the estimator approaches the true value of the parameter, as the sample grows, then it can be said that it is consistent. That is, if the amount of data tends to infinity then (θ_1) tends to Θ, e.g. Figure 16.

In accordance with the proposal to promote more confidence to DEP-DEA assessment modeling approach, incorporating some statistical treatment to perform goodness-of-fit test "R2" – the coefficient of determination – that be able to tell us how well the DP-DEA assessment line fit the data.

More generally, our interest is to predict the current values of dependents variables representative of a population, then the nature of the distribution of the errors $e_{ij} = (y_{ij} - \hat{y}_{i\theta j})$ needs to be assumed on behalf best fitted result to dataset extracted of some population.

The BUE achievement requires that each $e_{ij} = (y_{ij} - \hat{y}_{i\theta j})$ is distributed normally with: Mean: $E\,(e_{ij}) = 0$; Variance $E\,\left(e_{ij}^2\right) = \sigma^2$ and $\text{cov}\,(e_{ij}, e_{kj})$: $E(e_{ij}, e_{kj}) = 0 \;\; when\; i \; \neq k$

i.e.: $e_{ij} \sim N(0, \sigma^2)$ means that e_{ij} and e_{kj} are not only uncorrelated but normally and independently distributed (Gujarati 1995: 51-77, 191-237).

DP-DEA assessment modeling approach, in the same way of the confidence interval generated by ordinary least squares, instead of relying on a line estimator, an interval is defined around the estimator piece-wise line, e.g. see Figure 13. This interval “involves” the data with smaller or larger dispersion. On the other hand, the DP-DEA may also be shaped with the same goal: to adjust the upper and lower curves in the response variable (Gujarati 1995: 777-784).

7. History and relevance of Real Estate Evaluation Standards in Brazil

Brazilian Association of Technical Standards is a nonprofit organization that develops and publishes standards on a wide variety of topics, by the National Standardization Forum, composed of Committees (ABNT,CB) and Sectorial Standardization Organisms (ONS), which are in charge of the Brazilian Standards. Research reports are elaborated by delegates of the Study Commissions (ABNT,CE), composed of producers, consumers, users and other involved agents, such as universities and laboratories. The Committees (CB) and Organisms (ONS) thus develop a Project for the Standard, which circulates for Public Consultation amongst the ABNT’s associates and other interested parties.

Appraisal Engineering has one of the most main activity on the real estate valuation, prerogative for preserving the integrity of the mortgage lending process for the benefit of the general public and the parties involved (e.g. Banks, Promotion Agencies, Pension Funds) and the buyer (see Shapiro et al. 2013).

The real estate’s value estimation is drawn up by a civil engineer or architect, through the use of methodologies for the valuation of tangible (property, land holdings), intangible (commercial capital, industrial capital, brands, patents, etc.), and financial (shares, corporate rights) assets, established and regulated by the Brazilian National Uniform Standards of Professional Appraisal Practice – NBR – 14653 (ABNT 2011).

The first draft for real estate's appraisal standards organized by public entities and institutes directed to Appraisal Engineering date 1957. This was followed by others, of huge relevance, elaborated by institutes that operate in the field, but the subject acquired relevance during 60's great boom of expropriations, with the studies developed by commissions of professionals dedicated to specialized inspections and juridical assessments.

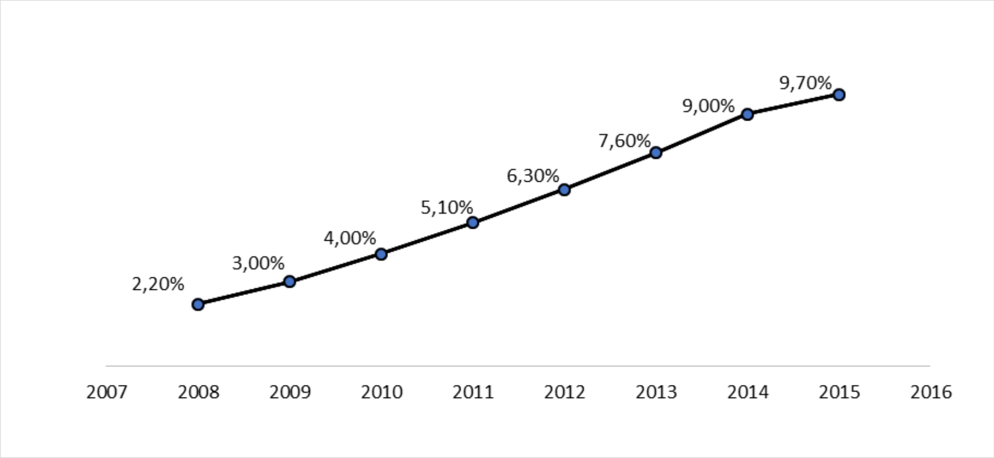
In 1977, emerges the first Brazilian Appraisal Norm of Urban Real Estate the ABNT's NBR 5676 (NB-502), whose major innovation is the establishment of accuracy levels for the evaluations. At the time, the ABNT started to produce other standards for evaluations, with the following typology: Rural Estates; Standardized Units; Industrial machines, equipment and Industrial complexes, and; Urban Glebes.

Revised in 1989, the Brazilian Appraisal Standards for Urban Real Estate is registered by the National Institute for Metrology (INMETRO) with # 5676. In this opportunity, the accuracy levels are transformed into stringency levels. The evaluation capacity of real estates was amplified and improved, along with its professionals, with modern statistical techniques in conjunction with information technologies. The important role given to the application of multiple linear regression in NBR 5676 of ABNT represented the initial stimulus for the appreciation of the use of data treatments based on scientific instrumental.

The basic assumptions of the use of statistical inference with the use of multiple linear regression started to stand out in the ABNT (2011) norm, drawn up in 2004, as a way of warning the evaluators of the damaging consequences of the non-observation of the adopted models and the consequent estimates. Fundamentally, the ABNT (2011) approaches the principles and methods that the appraisers shall attend, to ensure the necessary quality concerned in appraisal practices.

The Brazilian real estate market is in constant expansion, in view of public policies directed to increase the supply of real estates. There is a housing deficit in Brazil estimated in 10 million homes and a latent demand for more than 1.5 million homes per year, due to the insertion of young people in the labor market and related family formation. The increase of Mortgage Credit was launched year by year, that can be seen in Figure 17.

Figure 17. Mortgage Credit X GDP Brasil



Source: BACEN.

According to the specialized media, the Federal Saving Bank – CAIXA ECONÔMICA FEDERAL – is the major economic agent in the segment of mortgage credit, with a market-share of 67.2%, launching R$ 384,2 billion in 2015.

CAIXA is the biggest entity directly interested in incorporating the best practices and techniques in the daily real estate assessment's activities, a subcontracted service and contracts more than one million assessment reports Appraisers per year. The Department of Standards Management (GEPAD) has acted strongly, aiming to promote the research and incorporation of new technological advances in the Appraisal Engineering field and, when proved effective, the consolidation through ABNT standards.

The Assessment Engineering professionals are engineers and architects who become experts in this area through the implementation of specialized courses. The diffusion of the technique's improvement is disseminated via forums and congresses carried out in national and international levels. The SOBREA – Brazilian Appraisal Engineering Society of and the IBAPE – Brazilian Appraisal and Proficiency Institute of, with national representation in 21 federation units are the main entities bringing together the expertise of about 49,000 civil engineers and architects.

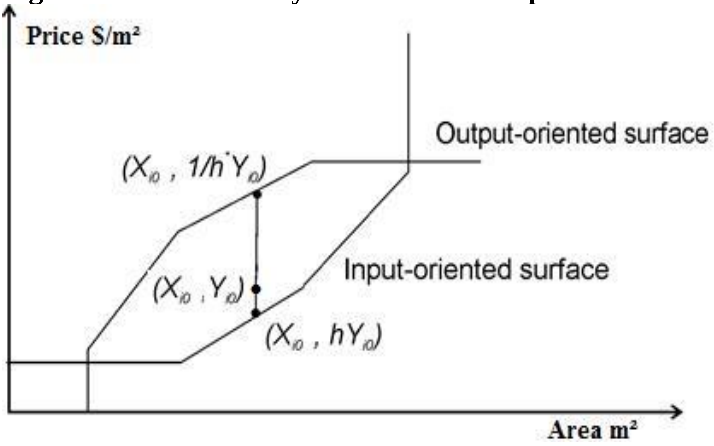
8. Regulatory and market context for applying the methodology in Brazil

The current fair market value is the objective of professional appraisal work, based on sample data collected into a local real estate market observed. A sample is fitted by similar sales with similar home's factors or features: style; dimensions, age, date, localization, influent sites and others. An appraiser has to observe a CMA (Comparative Market Analysis) (AI 2006,2013) that is analogues to the Direct Capital Comparison Method (DCCM) (ABNT 2011; Shapiro et al. 2013).

In Brazil, the NBR 14.653 – Brazilian Appraisal Norm of Urban Real Estate Appraisal relates the procedures to be observed by the professional appraisers in the activities regarded in works dealing by the stakeholders. One of the most important stakeholder that induce and foment the real estate market is the Federal Savings Bank – Caixa Economica Federal. Caixa has a great interest on configure the quality of appraisals works. With this proposal foment the diffusion of the knowledge into their contracted professionals and credential enterprises dialed to for fit appraisal works in 5,500 cities distributed in all Brazilian states.

Homogenizing the properties heterogeneity is done by the proceedings observed on ABNT (2011). NBR establish the procedure to translate observed factors and features properties into quality variables by allocated code, dummy variables or proxy variables. The quantitative variables like dimensions, age, distance and other factors observes its actual value. The more usual scientific method observed to appraisers is the MRA – Multiple Regression Analysis. Appraisers inferring observed prices of real estate market to assess market value of a specific property usually utilize several MRA software specifically designed. Complementary, the econometric model selected is the most properly fitted according to statistical criteria defined by the NBR and the MRA methodology available on Real Estate appraisal software tools.

Figure 18. DP-DEA Buyer and Seller Perspective Frontiers



At the same way, the extreme envelopment surfaces of DP-DEA method establish an interval for a property's value as a function of its physical features or factors and location proposed by Novaes (2002), see Figure 18. Lins et al. (2005: 3-8) describe the DP-DEA proceedings that makes use of two encapsulating surfaces that enfold, in an n-dimensional space, all the observed data, see Figure 5. Real estate units which, either from the point of view of the seller or the buyer, present an "efficient" price, will define those surfaces. The remaining units can have their value assessed by taking the envelopments as frameworks, under an output-oriented or an input-oriented DEA model.

The value of central tendency or "the fair market value" is describe by the surface performed by the combination of the attractive value estimated between the two encapsulating surfaces, which minimize the absolute deviation established by the center of mass or gravity of the whole distribution (see Novaes, Paiva 2010) and Figure 13.

9. Application of DP-DEA according based on the real estate market data.

Double Perspective DEA (DP-DEA) uses as an objective measure of the observed units' normalized distance to the two simultaneous perspectives: the hyper-plane by maximization of outputs and the minimization of inputs. In such a way that inputs under a buyer's perspective are the outputs under the seller's perspective and vice-versa, the dependent variable – apartment unit value ($,m²) (Lins et al. 2005: 3-14), see Figure 19.

DP-DEA proposes that, just as the seller wishes to maximize the output (selling value), given the inputs (property attributes), the buyer wishes to minimize its input (purchase value), given the outputs (property attributes). The problem is treated through two mathematical programming, which provides two DEA frontiers: the seller and the buyer, therefore the name DP-DEA given to the present method.

The DP-DEA method is usually applied to a database consisting of market values to the properties, which were established in scriptures of real estate transactions and that occurred in several regions of a city or on offerings. DEA production possibilities represent the scope for negotiation, containing the possible transaction values influenced by the supply or demand (see Shapiro et al. 2013), as a function of properties factors. Modeling starts by selecting" observed real estate data, with "m" independents variables, considering the usual main factors that report the characteristics property attributions and "s" dependent variable, mainly considering the property value and,or sale velocity, real estate build quality and others outputs of interest. DP-DEA model determines a subset composed of "k" data that belongs to the perspective's enveloping surface. These data are considered that the best attend one or other perspective and define the segments of the enveloping surface, thus motivating the envelope form of DEA CCR or BCC models (see Lins, Meza 2006). The contained subset, not belonging to this surface, is formed by the remaining "n – k" inefficient data to each perspective. The computation of the normalized distance of each observed unit requires the solution of a linear programming problem (Lins et al. 2005).

The LOOP,DP-DEA function (Novaes, Paiva 2010) appraisal the market value estimator between the two encapsulating surfaces, which minimize the median absolute deviation of the whole distribution.

DP-DEA establish a central tendency hyper-plane surface see section 6, see Figure 13. DP-DEA software estimates the dependent variables Φj by the minimization of the median absolute deviation of the whole distribution determined by one estimated escalate Z (Novaes, Paiva 2010). The LOOP,DP-DEA piece-wise line on R2 is illustrated in Figure 13, section 6, applying all possible variation of Φj (Novaes, Paiva 2010).

9.1. Practical example with a single Independent Variable.

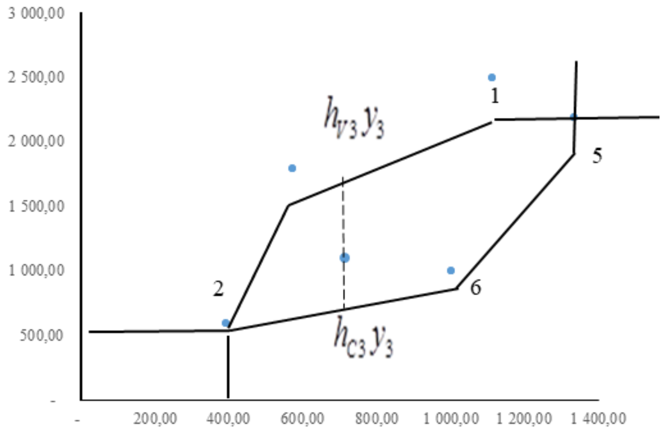
A practical approach of real estate appraisal based on urban lots sample with two variables. Data sample collected are defined in Table 4.

Table 4. Urban lots sample with two variables

				Transformed
Data	*Total Area*	*Unit Value*	*Total Value*	*1/ Total Area*
1	90.00	2,500.00	225,000.00	**1,111.11**
2	255.00	600.00	153,000.00	**392.16**
3	140.00	1,100.00	154,000.00	**714.29**
4	175.00	1,800.00	315,000.00	**571.43**
5	75.00	2,200.00	165,000.00	**1,333.33**
6	100.00	1,000.00	100,000.00	**1,000.00**

On Figure 19, R2 data graph allows visualizing double envelopment surfaces. The upper frontier is the locus of data highest value, the Seller's perspective frontier. According to the basic assumptions, item D.2.1 of Annex D of ABNT (2011), the transformation of the independent variable – Total Area – must be performed to present a positive correlation with the independent variable – Unit Value. In Table 4, the last column corresponds to the inverse function of the variable Total Area.

Figure 19. Total Area x Unit Value



The data set corresponds to the 2 x 6 matrix, where each column represents a lot. The 1st row corresponds to the independent variables and the 2nd row corresponds to the dependent variables:

$$\begin{bmatrix} 1111.11 & 392.16 & 714.29 & 571.43 & 1333.33 & 1000.00 \\ 2500.00 & 600.00 & 1100.00 & 1800.00 & 2200.00 & 1000.00 \end{bmatrix}$$

Decision variables are

h_{V0} and $\lambda = [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6]$

The value projected of each optic is estimated according to the projection of the data observed value through each envelope. According seller's viewpoint, the data "3" estimated value is equal:

$$y_{v3} = h_{V3} \times y_3$$

To estimate decision variable h_{V3} is used the Output-Oriented Model PPL VRS Envelopment as follows:

Envelopment (Dual) for BCC

$MaxH_{V0} = h_{V0}$

$$Subject to h_{v0} y_{r0} - \sum_{j=1}^{n} \lambda_j y_{rj} \leq 0, r = 1, \ldots\ldots, s$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{i0}, i = 1, \ldots\ldots, m$$

$$\lambda_j \geq 0, \forall k, j, i$$

A Linear Problem Programming to estimate the lot value “3” projected at the frontier of the seller’s perspective can be represented matricially as:

Envelopment (Dual) for BCC

$$Max [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \begin{bmatrix} h_{vo} \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{bmatrix}$$

Such that

$$\begin{bmatrix} 1100.00 & -2500.00 & -600.00 & -1100.00 & -1800.00 & -2200.00 & -1000.00 \\ 0 & 1111.11 & 392.16 & 714.29 & 571.43 & 1333.33 & 1000.00 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} h_{vo} \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{bmatrix} \begin{matrix} \leq \\ \leq \\ = \end{matrix} \begin{bmatrix} 0 \\ 714.29 \\ 1 \end{bmatrix}$$

$\lambda_j \geq 0, \forall k, j, i \; e \; h_v$ *irrestricted signal.*

The solution can be realized through the Simplex Method – Minimization Problem (Boldrini et al. 1980) or by Excel Solver. Through Performer DEA software (2016), we obtain the following results for the decision variables:

$$\begin{bmatrix} h_{vo} \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{bmatrix} = \begin{bmatrix} 0.5541 \\ 0.265 \\ 0 \\ 0 \\ 0.735 \\ 0 \\ 0 \end{bmatrix}$$

Seller's perspective, value estimated for data "3" is:

$y_{v3} = 1/h_{v3} \times y_3 = 1/0.5541 \times 1100.00 = 1985.30$[1]

The decision variable h_{C3} is estimated by Input-Oriented Model PPL VRS Envelopment (Lins et al. 2005), as follows:

Envelopment Problem (Dual)

$$MinH_{C0} = h_{C0}$$

$$Subject\ to\ \ h_{C0}\, y_{r0} - \sum_{j=1}^{n} \lambda_j y_{rj} \geq 0, \quad r = 1, \ldots\ldots, s$$

$$\sum_{j=1}^{n} \lambda_J x_{ij} \leq x_{i0}, \quad i = 1, \ldots\ldots, m$$

$$\lambda_j \geq 0, \forall k, j, i$$

$$ForVRS : \sum_{j=1}^{n} \lambda_j = 1$$

[1] *See Data "3" Table 5 – Maximum Value.*

From the data of the related Sample in Table 5, we obtain the following formulation:
Envelopment Problem (Dual)

$$Min\,[1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \begin{bmatrix} h_{co} \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{bmatrix}$$

Such that

$$\begin{bmatrix} 1100.00 & -2500.00 & -600.00 & -1100.00 & -1800.00 & -2200.00 & -1000.00 \\ 0 & 1111.11 & 392.16 & 714.29 & 571.43 & 1333.33 & 1000.00 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} h_{co} \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{bmatrix} \begin{matrix} \geq \\ \leq \\ = \end{matrix} \begin{bmatrix} 0 \\ 714.29 \\ 1 \end{bmatrix}$$

$$\lambda_j \geq 0, \ \forall\, k, j, i$$

The solution can be realized through the Simplex Method – Minimization Problem (Boldrini et al. 1980) or by Excel Solver. Through Performer DEA software (2016), we obtain the following results for the decision variables:

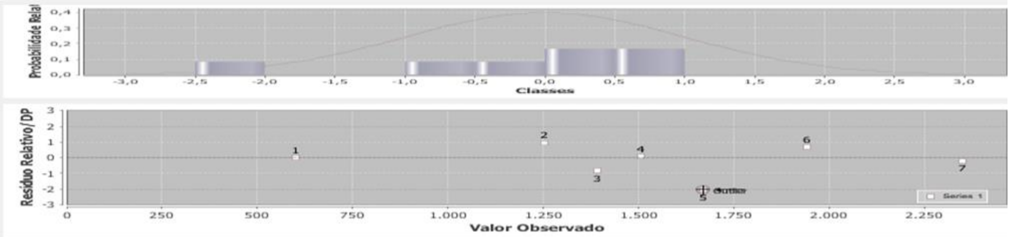
$$\begin{bmatrix} h_{co} \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{bmatrix} = \begin{bmatrix} 0.7382 \\ 0 \\ 0.43 \\ 0 \\ 0 \\ 0 \\ 0.57 \end{bmatrix}$$

According buyer's viewpoint, the data "3" estimated value is equal to: $y_{C3} = h_{C3} \times y_3 = 0.7382 \times 1100.00 = R\$812.00$[2]

Data "3" estimated the fair market value, according equations (24) and (28) (Novaes, Paiva 2010) is R$1,390.31[3].

Table 5. Minimum, Maximum and Estimated Values – DP-DEA

Data	Observed Value	Minimum Value	Estimate Value	Maximum Value	Residuals	Relative Residuals	Relative
1	2.500	1.400	1.942,193	2.500	557,807	22,31%	0,69
2	600	600	600	600	0	0,00%	0,00
3	1.100	811,982	1.390,311	1.985,294	-290,311	-26,39%	-0,81
4	1.800	717,972	1.251,307	1.800	548,693	30,48%	0,94
5	2.200	2.200	2.347,871	2.500	-147,871	-6,72%	-0,21
6	1.000	1.000	1.668,318	2.355,882	-668,318	-66,83%	-2,05
7	1.545,292	890,323	1.506,147	2.139,706	39,145	2,53%	0,08



9.2. DP-DEA software graph analysis

Table 5 reports a simulating of real estate appraisal data. Data "7" is characterized on Table 6.

Figure 20 reports observed, central tendency, minimum and maximum estimate values. The data on abscissa axis and unit value on ordinate axis.

[2] *See Data "3" Table 5 – Minimum Value.*
[3] *See Data "3" Table 5 – Estimated Value.*

Table 6. Urban Lots Sample – Area x Unit Value – Appraising Data “7”

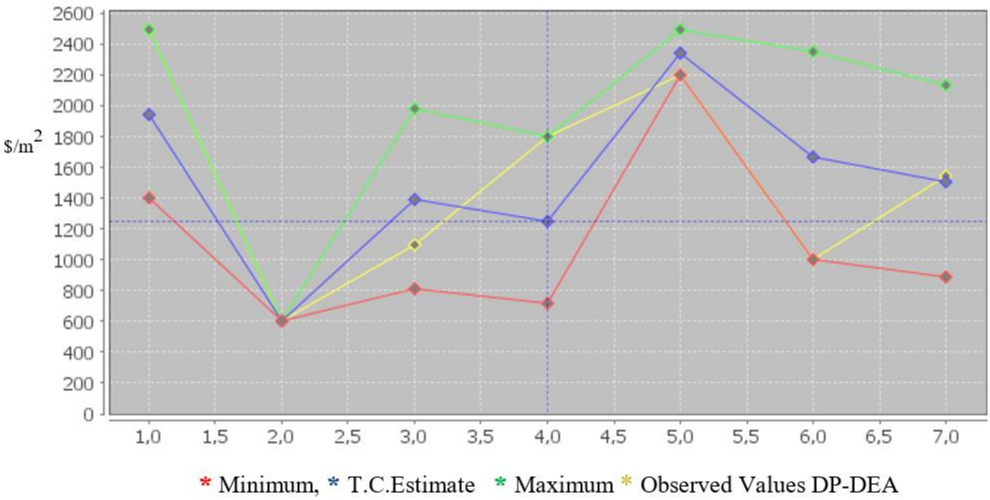
	A	E	Address	Complement	Privative Area	Unit price	Total Price
1	☑	☐	R. Ipê	15	90,00	2500,00	225000,00
2	☑	☐	R. Azaleia	223	255,00	600,00	153000,00
3	☑	☐	R. Bromelia	165	140,00	1100,00	154000,00
4	☑	☐	R. Jasmim	33	175,00	1800,00	315000,00
5	☑	☐	R. Camelia	29	75,00	2200,00	165000,00
6	☑	☐	R. Margarida	68	100,00	1000,00	100000,00
7	☑	☑	Dado Novo	a estimar	120,00		

Data “7” estimate Value with area of 120.00 m2:

Tendency Central R$

Minimum Value R$ Maximum R$

Figure 20. Minimum, estimated, maximum and observed Values DP-DEA



9.3. DP-DEA software modeling and application on real estate appraisal

DP-DEA software incorporates some statistical and graphs analysis: Dependent variables x Independent variables; coefficient of determination (R^2); multiple correlation (R); correlation analysis; Table of Residual Standard Deviation and Graph Analysis; Graph of appraisal values and observed, buyer and seller’s perspective values variations; Graph of adherence between observed and assessed variables. The DP-DEA software is an innovator product that observes the Double Perspective Data Envelopment Analysis Methodology.

Real estate appraisal is the action of judging the market value of properties – lands and its improvements. The market value improves an important role in land management involving public administration, institutions of landing regulations, properties financial institutions and so, a broad range of stakeholders, until meet the personal interest of be a property owner. Then the market value would be comprehensive and reliable enough to attend all these actors. At the end of a sale process, we can represent all these actors by the two involved negotiator the buyer and the seller.

The DP-DEA approach by the Direct Capital Comparison (DCC) (Mackmin 1994) method was applied to a database consisting of the market values of apartments that were established in transactions, and which have occurred in several neighborhoods of the Niteroi's municipal district of Rio de Janeiro's state.

In the predictive modeling procedures, we need to establish a sample using information extract of some Real Estate agent's data. Nowadays, the internet enables to get complementary information on Real Estate's sites. Websites allows one to either send a lead inquiry or search through property type categories: eventual sale price, location, styling, sale condition, size, amenities, features, improvements and upgrades, attractive or economic sites, the current real estate market and mortgage interest rates, among others.

The parameterization consists in depict subjective amenities or property characteristics in quantitative or qualitative variables. This phase is one of the most important that involves the modeling process, is essential to adequate properly the scales and variations of each qualitative variable. It can be configured by allocate code, dummy variable and proxy variable.

The database used in DP-DEA computations consists of 14 variables (features) for each one of the 76 residential units (apartments). These variables include: buy and sell transaction price; property privative area; Effective age; number of rooms, baths, bedrooms; building amenities; parking places; Quality of Construction; neighborhood index and unit's conservation; date of collecting data, corresponding event. For the description and comments of each one sees U.S. Department of Housing and Urban Development (Housing Department, 2015), Appraisal Institute Commercial Data Standards (2004) or Brazilian National Uniform Standards of Professional Appraisal Practice (ABNT, 2011).

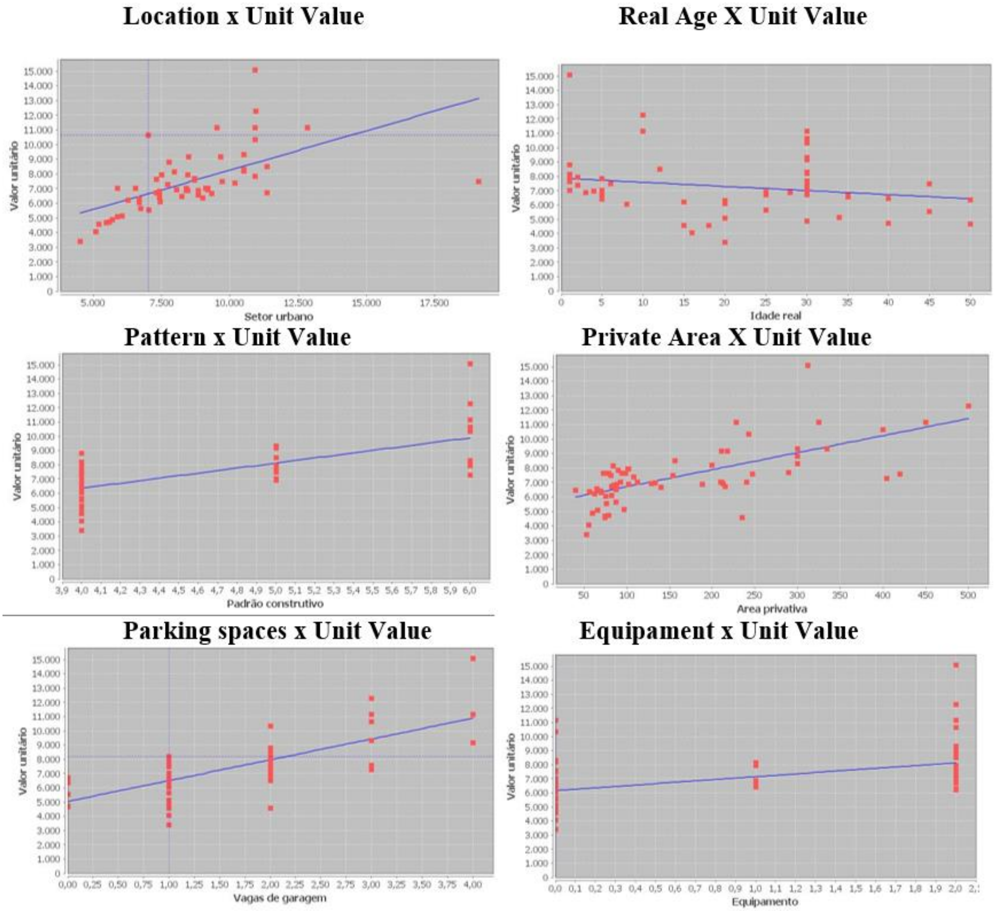
First, we proceed an exam to understand the variables behavior and correlation, see Table 7 and Figure 21.

Table 7. Variables Correlation Analysis

Variable	Event	Month	Age	Conserve	Pattern	Equipment	Private Ar	Parking Sp	Bedroom	Bath	Serv room	Location	Panor View	Unit value
Event	100,00%	42,79%	35,28%	-11,98%	35,83%	7,87%	51,12%	32,58%	38,18%	39,42%	-4,20%	37,72%	39,49%	37,95%
Month	42,79%	100,00%	3,42%	-11,94%	34,78%	28,52%	60,96%	59,50%	48,35%	51,35%	-5,81%	29,33%	46,93%	31,67%
Age	35,28%	3,42%	100,00%	-59,32%	-16,90%	-47,99%	18,12%	-10,37%	-1,24%	-3,59%	-4,41%	11,60%	14,51%	-6,96%
Conservation	-11,98%	-11,94%	-59,32%	100,00%	21,11%	23,19%	-10,88%	4,02%	-1,04%	2,02%	-4,03%	-9,71%	-12,72%	23,42%
Pattern	35,83%	34,78%	-16,90%	21,11%	100,00%	29,60%	47,87%	59,08%	44,47%	51,85%	38,05%	40,57%	36,53%	55,28%
Equipment	7,87%	28,52%	-47,99%	23,19%	29,60%	100,00%	30,70%	39,23%	28,62%	36,35%	5,55%	18,32%	5,12%	15,66%
Private Area	51,12%	60,96%	18,12%	-10,88%	47,87%	30,70%	100,00%	73,58%	75,12%	76,13%	21,91%	43,19%	62,96%	34,49%
Parking Space	32,58%	59,50%	-10,37%	4,02%	59,08%	39,23%	73,58%	100,00%	76,06%	76,06%	34,36%	41,91%	49,56%	48,43%
Bedroom	38,18%	48,35%	-1,24%	-1,04%	44,47%	28,62%	75,12%	76,06%	100,00%	70,94%	20,66%	45,39%	42,75%	29,91%
Bath	39,42%	51,35%	-3,59%	2,02%	51,85%	36,35%	76,13%	76,06%	70,94%	100,00%	22,64%	37,64%	46,13%	46,76%
Service room	-4,20%	-5,81%	-4,41%	-4,03%	38,05%	5,55%	21,91%	34,36%	20,66%	22,64%	100,00%	16,52%	17,36%	24,63%
Location |	37,72%	29,33%	11,60%	-9,71%	40,57%	18,32%	43,19%	41,91%	45,39%	37,64%	16,52%	100,00%	30,59%	32,35%
Panoramic View	39,49%	46,93%	14,51%	-12,72%	36,53%	5,12%	62,96%	49,56%	42,75%	46,13%	17,36%	30,59%	100,00%	53,64%
Unit value	37,95%	31,67%	-6,96%	23,42%	55,28%	15,66%	34,49%	48,43%	29,91%	46,76%	24,63%	32,35%	53,64%	100,00%

The variables of the apartments collected have the following configuration:
Event – if sold or sale pending – dummy variable;
Month – first is 20th January;
Actual Age;
Conservation – allocated code – scale 1 to 6 representing the first the necessary recuperation to new (first use);
Building Class – allocated code – minimum class is one and high class = 6;
Building Amenities – allocated code – without amenities is zero; playground = 2, and swimming pool = 3;
Parking Space – quantitative variable;
Private area – square meters – number of parking places;
Location – proxy variable by familiar income on Census Tract geocode map;
Bedrooms – quantitative variable;
Bathrooms – quantitative variable;
Service rooms – quantitative variable;
Panoramic view – yes or no – dummy variable;
Unit value – R$,m².

Figure 21. Relation between dependent and independent variables



Relationship view between dependent variable e each on independent variable of the six selection, e.g., see Figure 21. Will permit see each data adherence with the data set tendency line. Putting on suspicion the data not adequate.

First model graph analysis simulation

The correlation analysis presented in Table 8 is the result of an optimization procedure, to obtain a best relation sheep by transforming the independent variables.

Table 8. Transformed Variables Correlation Analysis – 1 step

#	Variáveis	Ativa?	T. Imposta?
1	Variables	☐	x
2	Event	☐	x
3	Month	☑	$1/x^2$
4	Age	☐	x
5	Conservation	☑	x^2
6	Pattern	☑	x^2
7	Equipment	☑	x
8	Private Area	☑	x
9	Parking Space	☐	x
	Bedroom		

Opções: $1/x$, $1/x^2$, x^2, ln(x), $\sqrt{x}$, $1/\sqrt{x}$ — Todos — Nenhum — Inverter

#	Form	Variable	Age	Pattern	Equipment	Privte Area	Park Space	Location	Pano View	Unit Value
1										
2										
3	x^2	Age	37,67%	27,44%	100,00%	31,70%	38,75%	27,02%	6,08%	15,82%
4	x	Pattern	-17,78%	48,78%	31,70%	100,00%	73,58%	46,20%	62,96%	34,49%
5	x	Equipment	-10,24%	59,24%	38,75%	73,58%	100,00%	46,82%	49,56%	48,43%
6	ln(x)	Privte Area	-9,77%	41,74%	27,02%	46,20%	46,82%	100,00%	32,67%	37,53%
7	x	Park Space	-15,16%	37,50%	6,08%	62,96%	49,56%	32,67%	100,00%	53,64%
8	x	Location	10,56%	55,62%	15,82%	34,49%	48,43%	37,53%	53,64%	100,00%
		Pan View								
		Unit Value								

Correlação — DEA — Cancelar

The modeling process are implemented by graph analysis, it permits understanding the relationship between variables, its correlation, see Table 8. By DP-DEA software simulating with all variables and dataset, we concluded that is necessary to extract 6 outliers' data or with Relative Residuals more than 35%. Correlation analysis enable selecting the variable with highest explanation with the dependent variable Transformed independent variables to be more adjusted with market behavior.

Figure 22. Relative probability distribution of residuals comparing fitness adherence with normal distribution

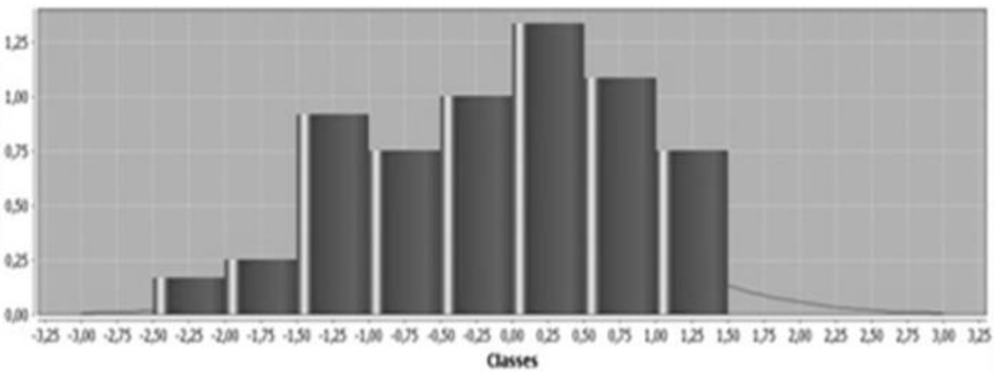


Figure 23. Relative Residuals , Pattern Deviation Outliers Analysis and Homoscedasticity

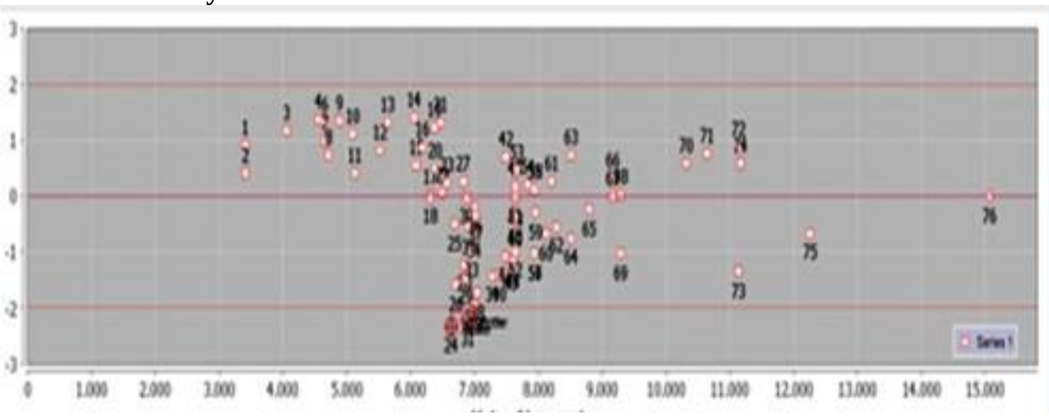
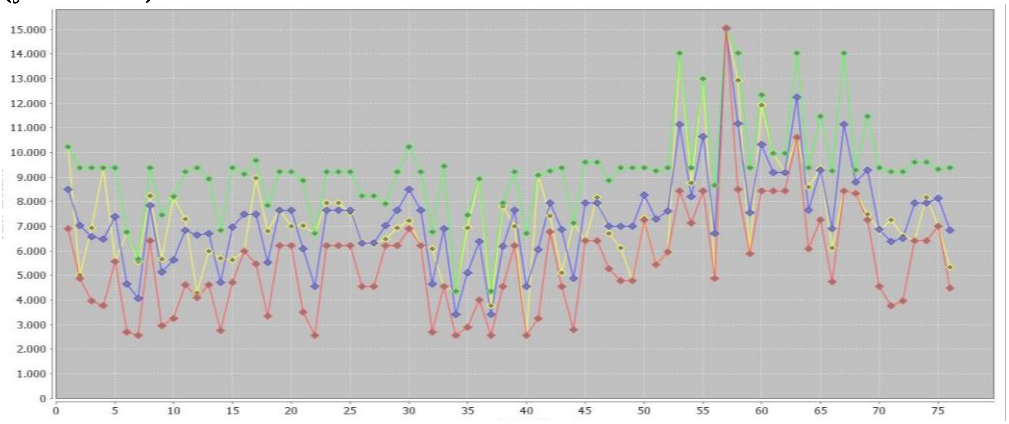


Figure 24. Appraisals of unit value (blue line) of central tendency (loop DEA), buyer's (red line) and seller's value (green line) and observed value data (yellow line)



* Minimum, * T.C.Estimate * Maximum * Observed Values DP-DEA

Figure 25. Adherence between Unit Value Observed and Appraisal Values

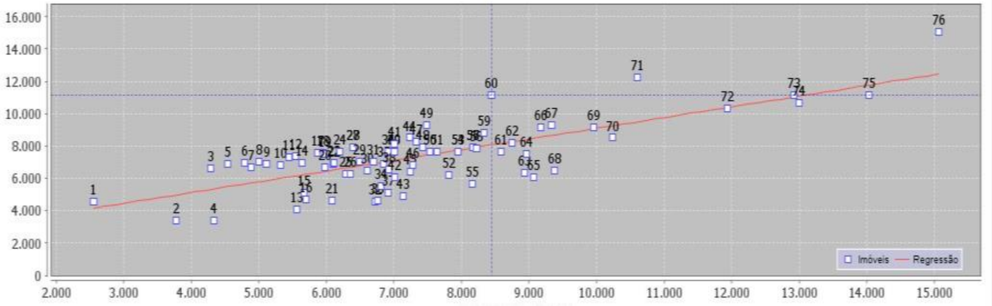
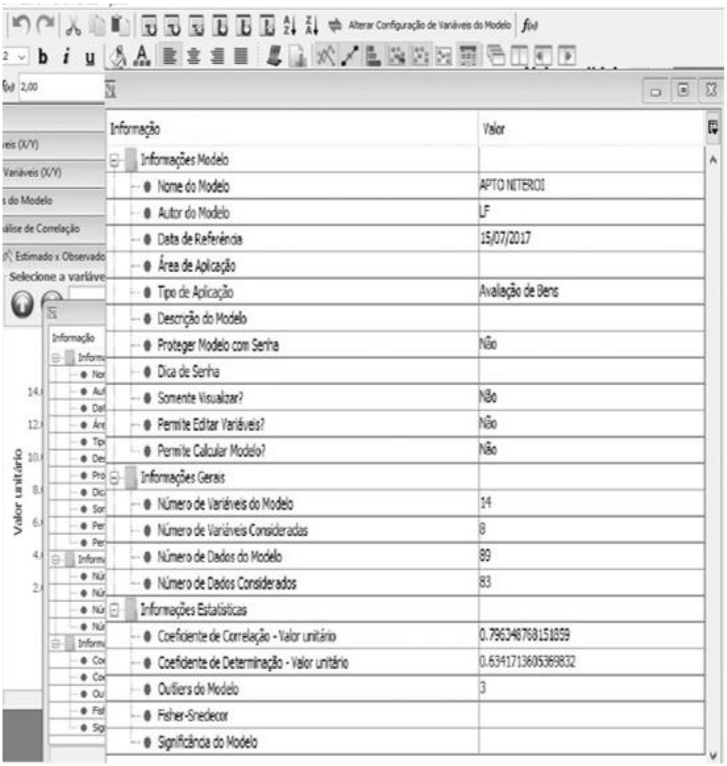


Figure 26. Model analysis R²



R² model analysis, see Figure 26. Results: R^2 = 63.42%; R = 79.63%; Outliers = 3; Data = 83; Variable = 8.

Analyzing if the DP-DEA appraiser model is adequate. Steps:

Graph in Figure 22 – Verifying Fitness adherence with Normal distribution.
Graph in Figure 23 – Examining the residuals distribution to verify homoscedasticity.
Graph in Figure 24 – The adjust of distance between seller's and buyer's frontiers.
Graph in Figure 25 – Verifying the adherence between observed and appraisal values.
Table in Figure 26 – The first model analyzed has $R^2 = 0.6341$
Conclusion of this first model simulation: We conclude that some analysis criteria doesn't corresponding with expected results. We decide to continue the process to find a better model.

9.4. Real Estate appraisal DP-DEA model selected

After testing and simulating some models, we stopped when the model achieved the following results: Verifying the correlation between the variables selected if presented the expected behavior with the predicted in real estate market

Table 9. Relation between dependent and independent selected variables

Análise de Correlação

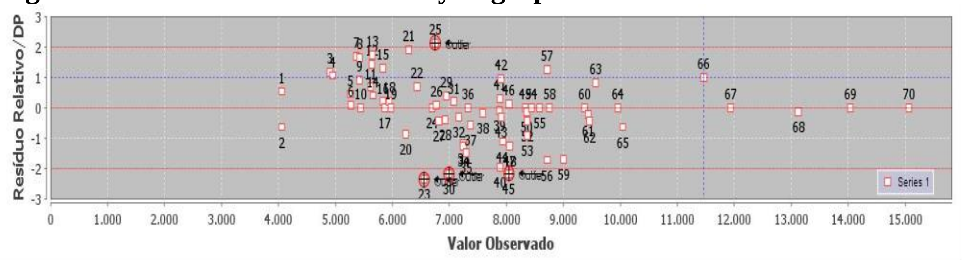
Selecione a variável que deseja-se analisar

Variável: Idade real

#	Forma	Variável	Idade real	Padrão cons...	Equipa...	Area pri...	Vagas de g...	Setor Cen...	Vista pano...	Valor u...
1	$1/x^2$	Actual Age	100,00%	9,63%	40,64%	8,97%	-11,13%	-10,83%	-17,09%	7,30%
2	x^2	Quality Building	9,63%	100,00%	28,48%	-50,85%	61,68%	42,86%	40,21%	48,90%
3	x^2	Equipments	40,64%	28,48%	100,00%	-35,98%	40,61%	25,33%	2,74%	18,31%
4	1/x	Privative Area	8,97%	-50,85%	-35,98%	100,00%	-71,74%	-61,21%	-57,80%	-31,68%
5	x	Parking Space	-11,13%	61,68%	40,61%	-71,74%	100,00%	50,09%	53,43%	46,89%
6	ln(x)	Location	-10,83%	42,86%	25,33%	-61,21%	50,09%	100,00%	36,99%	32,80%
7	x	Panoramic View	-17,09%	40,21%	2,74%	-57,80%	53,43%	36,99%	100,00%	43,63%
8	x	Unit Value	7,30%	48,90%	18,31%	-31,68%	46,89%	32,80%	43,63%	100,00%

Verifying Data Residuals Outliers and Homoscedasticity, see Figure 27.

Figure 27. Outliers and errors analysis graph



Verifying fitness adherence of Residuals with Normal Distribution (Figure 28); adherence between observed and appraisal values (Figure 29); interval amplitude between the buyer and seller frontiers (Figure 30).

Figure 28. Fitness adherence with normal distribution

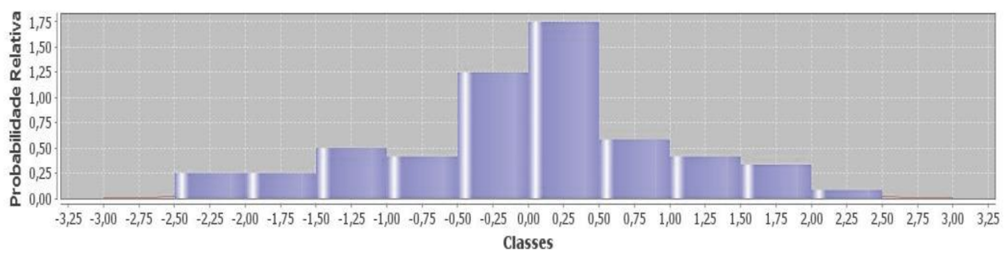


Figure 29. Adherence between observed and appraisal values

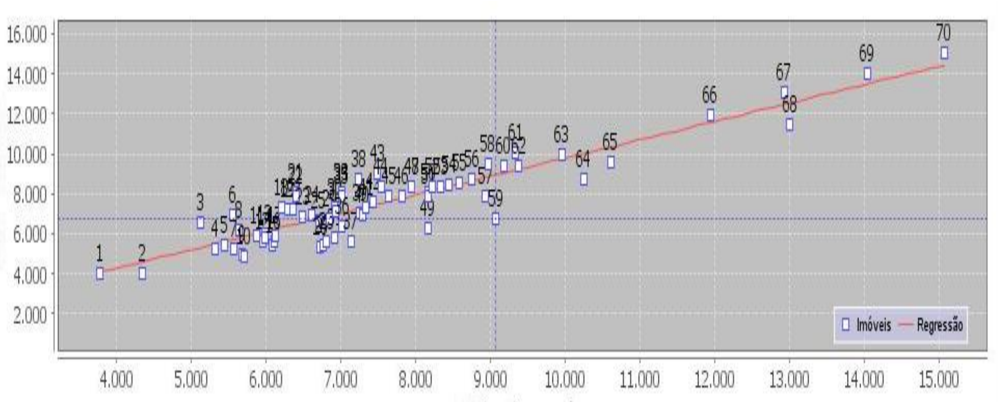
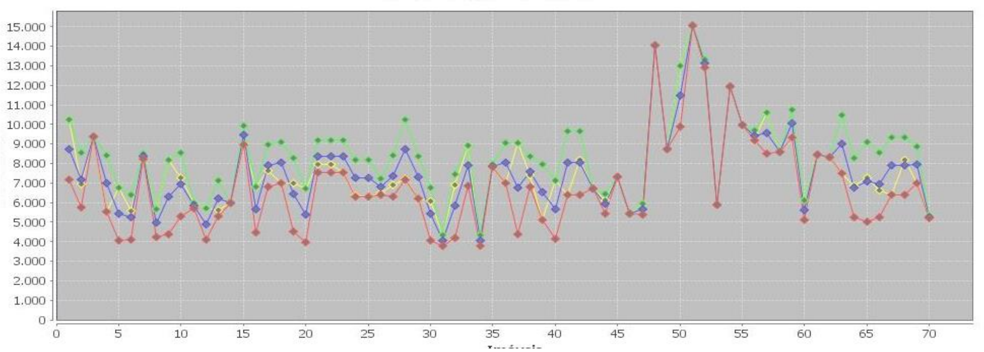
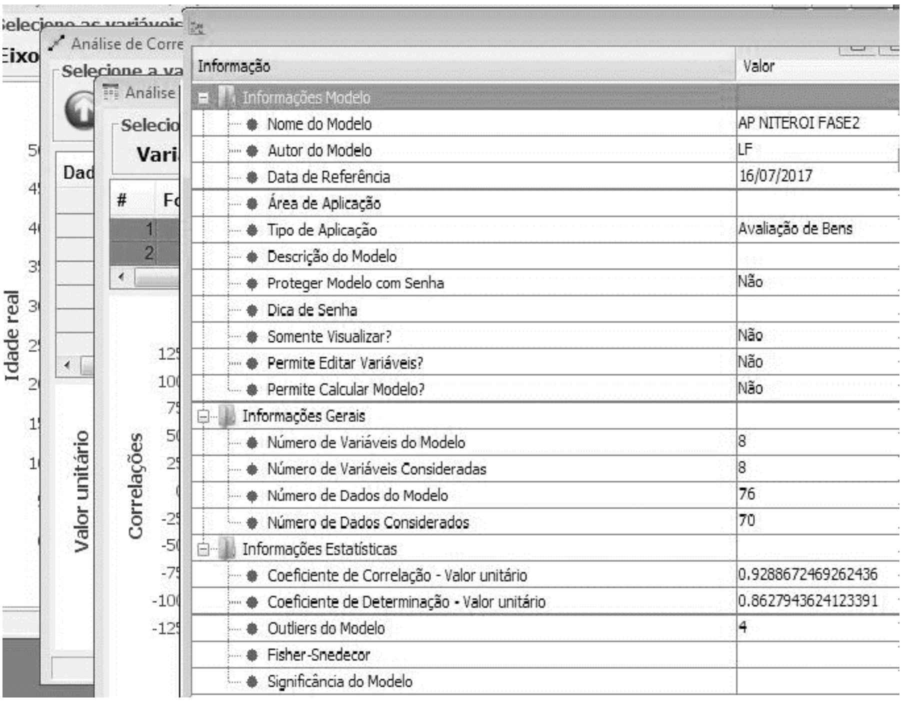


Figure 30. Appraisal values (blue line) of Central Tendency (Loop DEA), Buyer's (red line) and Seller's values (green line) and Observed data values (yellow line)



* Minimum, * T.C.Estimate * Maximum * Observed Values DP-DEA

Figure 31. R^2



Verifying R^2 Model Analysis.

9.5.The Selected model

This last model selected achieves the appraised values, in tables 10 and 11, were, fundamentally, validated by the statistical analysis, it was achieved by model that best observes the BUE assumptions. This last model is constituted by transforming some of independent variables defined in Table 7. It shows best statistical parameters obtained comparatively with others simulated models: expected influence of independent variables with dependent variable; best R^2 = 0.86279; residuals fitness adherence; errors with normal distribution and outliers' results; smaller amplitude between buyer and seller frontier; better adherence between assessed and observed values of dataset, see figures 27, 28, 29, 30 and 31.

Real estate appraisal of central tendency, buyers and sellers' perspectives valuation and residuals statistical results for observed and appraisal data are related in tables 10 and 11.

Table 10. Results of DP-DEA software – Assessed and Projected Values and Residuals

Data	Value Observed	Buyer Perspective Value	Assess Value	Seller Perspective Value	Residuals	Relative Residuals	Relative Residual,Standard Desviation
1	10,239.34	7,164.54	8,716.55	10,239.34	1,522.79	14.87%	1.25
3	6,923.08	5,740.41	7,167.09	8,566.90	- 244.01	-3.52%	-0.3
4	9,375.00	9,375.00	9,375.00	9,375.00	-	0.00%	0
5	5,546.31	5,546.31	6,992.29	8,411.04	- 1,445.98	-26.07%	-2.2
6	6,756.76	4,068.94	5,425.62	6,756.76	1,331.14	19.70%	1.66
7	5,563.64	4,102.25	5,264.22	6,404.30	299.42	5.38%	0.45
8	8,222.22	8,222.22	8,358.68	8,492.57	- 136.46	-1.66%	-0.14
9	5,670.10	4,219.15	4,951.52	5,670.10	718.58	12.67%	1.07
10	8,160.92	4,384.78	6,290.79	8,160.92	1,870.13	22.92%	1.93
11	7,282.98	5,307.73	6,944.39	8,550.23	338.59	4.65%	0.39
13	5,975.61	5,694.73	5,836.50	5,975.61	139.11	2.33%	0.2
14	5,696.20	4,095.20	4,903.31	5,696.20	792.89	13.92%	1.17
15	5,639.10	5,313.18	6,230.39	7,130.34	- 591.29	-10.49%	-0.88
16	5,974.03	5,974.03	5,974.03	5,974.03	-	0.00%	0
17	8,964.76	8,964.76	9,455.01	9,936.03	- 490.25	-5.47%	-0.46

Table 10. Cont …

Data	Value Observed	Buyer Perspective Value	Assess Value	Seller Perspective Value	Residuals	Relative Residuals	Relative Residual,Standard Desviation
18	6,806.66	4,449.77	5,639.41	6,806.66	1,167.25	17.15%	1.45
19	7,640.39	6,824.30	7,908.00	8,971.29	- 267.61	-3.50%	-0.3
20	7,003.69	7,003.69	8,056.72	9,089.93	- 1,053.03	-15.04%	-1.27
21	7,012.20	4,534.76	6,432.54	8,294.59	579.66	8.27%	0.7
22	6,717.72	3,985.22	5,364.45	6,717.72	1,353.27	20.14%	1.7
23	7,945.21	7,534.25	8,366.09	9,182.26	- 420.88	-5.30%	-0.45
24	7,945.21	7,534.25	8,366.09	9,182.26	- 420.88	-5.30%	-0.45
25	7,534.25	7,534.25	8,366.09	9,182.26	- 831.84	-11.04%	-0.93
26	6,357.14	6,285.71	7,244.94	8,186.11	- 887.80	-13.97%	-1.18
27	6,285.71	6,285.71	7,244.94	8,186.11	- 959.23	-15.26%	-1.29
28	6,487.19	6,397.48	6,812.86	7,220.42	- 325.67	-5.02%	-0.42
29	6,914.89	6,310.23	7,370.61	8,411.04	- 455.72	-6.59%	-0.56
30	7,231.54	7,164.54	8,716.55	10,239.34	- 1,485.01	-20.54%	-1.73
31	6,196.58	6,196.58	7,291.02	8,364.86	- 1,094.44	-17.66%	-1.49
32	6,081.08	4,068.94	5,425.62	6,756.76	655.46	10.78%	0.91
34	4,339.62	3,773.58	4,059.29	4,339.62	280.33	6.46%	0.54
35	6,914.89	4,179.66	5,828.69	7,446.68	1,086.20	15.71%	1.32

Table 11. Results of DP-DEA software – Appraisal and Projected Values and Residuals

Data	Value Observed +-	Buyer Perspecti-ve Value	Appraisal Value	Seller Perspective Value	Residuals	Relative Residuals	Relative Residual, Standard Deviation
36	8,928.57	6,863.40	7,905.80	8,928.57	1.022.77	11.46%	0.97
37	3,773.58	3,773.58	4,059.29	4,339.62	- 285.71	-7.57%	-0.64
38	7,808.76	7,808.76	7,880.48	7,950.84	- 71.72	-0.92%	-0.08
39	7,005.25	7,005.25	8,047.60	9,070.33	- 1.042.35	-14.88%	-1.25
41	9,066.67	4,393.54	6,752.31	9,066.67	2.314.36	25.53%	2.15
42	7,425.74	6,805.44	7,585.35	8,350.57	- 159.61	-2.15%	-0.18
43	5,113.64	5,113.64	6,555.28	7,969.77	- 1.441.64	-28.19%	-2.38
44	7,133.33	4,128.33	5,645.11	7,133.33	1.488.22	20.86%	1.76
45	6,395.12	6,395.12	8,044.83	9,663.47	- 1.649.71	-25.80%	-2.17

Table 11. Cont. …

Data	Value Observed +-	Buyer Perspective Value	Appraisal Value	Seller Perspective Value	Residuals	Relative Residuals	Relative Residual, Standard Deviation
46	8,166.08	6,395.12	8,044.83	9,663.47	121.25	1.48%	0.13
47	6,696.43	6,696.43	6,696.43	6,696.43	- 0.00	0.00%	0
48	6,113.74	5,420.47	5,945.35	6,460.36	168.39	2.75%	0.23
50	7,333.33	7,319.35	7,326.41	7,333.33	6.92	0.09%	0.01
51	5,445.54	5,445.54	5,445.54	5,445.54	0.00	0.00%	0
52	5,952.38	5,365.68	5,661.82	5,952.38	290.56	4.88%	0.41
53	14,035.08	14,035.08	14,035.08	14,035.08	-	0.00%	0
54	8,750.00	8,750.00	8,750.00	8,750.00	0.00	0.00%	0
55	13,000.00	9,899.99	11,464.73	13,000.00	1.535.27	11.81%	1
57	15,064.10	15,064.10	15,064.10	15,064.10	-	0.00%	0
58	12,923.07	12,923.07	13,121.72	13,316.63	- 198.65	-1.54%	-0.13
59	5,870.44	5,870.44	5,870.44	5,870.44	- 0.00	0.00%	0
60	11,934.15	11,934.15	11,934.15	11,934.15	0.00	0.00%	0
61	9,952.60	9,952.60	9,952.60	9,952.60	0.00	0.00%	0
62	9,174.31	9,174.31	9,436.87	9,694.48	- 262.56	-2.86%	-0.24
63	10,600.00	8,503.35	9,561.64	10,600.00	1.038.36	9.80%	0.83
64	8,581.31	8,581.31	8,581.31	8,581.31	- 0.00	0.00%	0
65	9,333.33	9,333.33	10,041.95	10,737.23	- 708.62	-7.59%	-0.64
66	6,103.28	5,113.64	5,613.16	6,103.28	490.12	8.03%	0.68
67	8,444.44	8,444.44	8,444.44	8,444.44	-	0.00%	0
68	8,333.33	8,333.33	8,333.33	8,333.33	-	0.00%	0
69	7,485.02	7,485.02	9,000.10	10,486.65	- 1.515.08	-20.24%	-1.71
70	6,847.84	5,232.64	6,769.21	8,276.85	78.63	1.15%	0.1
71	7,246.38	5,016.05	7,076.08	9,097.31	170.30	2.35%	0.2
72	6,606.68	5,243.27	6,923.11	8,571.32	- 316.43	-4.79%	-0.4
73	6,395.12	6,395.12	7,887.45	9,351.67	- 1.492.33	- 3.34%	-1.97
74	8,166.08	6,395.12	7,887.45	9,351.67	278.63	3.41%	0.29
75	7,005.99	7,005.99	7,937.10	8,850.68	- 931.11	-13.29%	-1.12
76	5,319.15	5,224.15	5,272.10	5,319.15	47.05	0.88%	0.07

10. Conclusion

With this paper we expect to open new paths that can be explored by DEA community. DP-DEA method is a useful tool to assess either performance or asset value when we need to consider two divergent interests but that needs to fulfill a common result. The statistical analysis incorporated on this appraisal application demonstrates its accuracy, achieving best-fit results and fulfilling a legal rule established on the Brazilian Professional Appraisal Practice Standards.

We would like also to emphasize that DP-DEA software appraisal capacity has innovated DEA study area, as an appraisal tool. In Table 12 – Simulation Results of DP-DEA Appraisal software, we present a simulation that considers the value appraisal of six different apartments. Modifying the two elected factors that influence the apartment valuation (Privative Area (m²) and Actual Age) and keeping the others factors constant (Building Quality, Equipment, Parking Space and Panoramic View), DP-DEA software could find the three important values: buyer perspective valuation, seller perspective valuation and central tendency, according to real market behavior.

Table 12. Simulation Results of DP-DEA software – Appraisal

	Actual Age Year	Quality Building	Equip a-ment	Privative Area m²	Parking Space	Location $, m²	Pano rami c View	Buyer Perspective $, m²	Value Assessed $, m²	Perspective Value $, m²
Apartment 01	1	5	1	150	2	9719.85	1	11,389.00	**13,209.09**	15.064.10
Apartment 02	15	5	1	80	2	9719.85	1	8,198.78	**9,836.50**	11.384.76
Apartment 03	15	5	1	150	2	9719.85	1	8,198.78	**9,714.63**	11.201.93
Apartment 04	15	5	1	300	2	9719.85	1	7,.062.96	**7,746.05**	8.391.83
Apartment 05	15	5	1	160	2	9719.85	1	8,.198.78	**9,707.10**	11.187.01
Apartment 06	30	5	1	150	2	9719.85	1	8,198.78	**9,705.53**	11.183.90

Other important analytic capacity of DP-DEA software goes far beyond, DP-DEA (Novaes 2002; Lins et al. 2005) making it possible to understand each perspective behavior (buyer , seller) through the mathematical Multiplier DEA Model frontier formulation. A practical approach inferring the buyer behavior, was presented on section 9. On the related example, it was considered the combination of factors that motivate the decision to buy a real estate in a specific location. The capacity of the parameters to create the frontier hyper plane, defined by the constraints of DP-DEA Input-oriented Model (Novaes 2002), illustrates the real estate buyer's behavior, like price-elasticity, verified on lower DP-DEA frontier. Each real estate price-elasticity graph bar is geocoded on Figure 1 – GIS representation of physical features of apartments appraisal of competitive buyer's perspective. Secondly, we used Loop-DP-DEA (Novaes, Paiva 2010) to find the central tendency, considering a sample of the market data set. Complementary, in section 9, we have demonstrated the fair market value appraisal incorporating statistical analysis to search the best unbiased estimator DP-DEA model.

The main contribution of this paper, considering estimation techniques assumptions, is demonstrated in section 9.4 and 9.5, concluding that DP-DEA modeling approach attends the assumption of Best Unbiased Estimator, Koch (2013). Thus, through the successive iterations of DP-DEA, we could obtain the minimization of estimation's mean square errors and the expected value of the error tending to zero.

The most usual technique used for appraisals is MLRA, which adopters resist to switch to other techniques, because it is already a consolidated methodology. Considering this scenario, we are making a huge effort to spread this new technique and to incorporate consistent statistical analysis to improve the estimation accuracy, when compared to the real dimension prospected.

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